Enrollment No./Seat No.:

GUJARAT TECHNOLOGICAL UNIVERSITY

Bachelor of Engineering - SEMESTER - IV EXAMINATION - WINTER 2025

Subject Code: 3140708 **Date: 20-11-2025**

Subject Name: Discrete Mathematics

Time: 02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Draw the graph with 4 nodes and 7 edges.	**03**
	(b)	Define simple graph, degree of a vertex, finite and infinite graphs and complete graph.	**04**
	(c)	(i) If A = B = {1, 2, 3, 5, 6, 10, 15, 30} and relation R defined as a/b, where a∈A and b∈B. Find the relation matrix. (ii) Solve the recurrence relation $a_n - 3a_{n-1} = 2, n \geq 1, a_0 = 1$.	**07**
Q.2	**(a)**	Let A, B and C be the sets such that $(A \cap B \cap C) = \emptyset$, $(A \cap B) \neq \emptyset$, $(A \cap C) \neq \emptyset$, $(B \cap C) \neq \emptyset$. Draw the corresponding venn diagram.	**03**
	(b)	Define subgroup of a group. Also, show that the subset H of a group of integers I is a subgroup under addition, where H = {...., -2m, -m, 0, m, 2m, ...}.	**04**
	(c)	How many integers between 1 to 2000 are divisible by 2, 3, 5 or 7.	**07**
		OR	
	(c)	Using venn diagram, prove the following (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$	**07**
Q.3	**(a)**	Check whether the proposition $((p \vee q) \wedge \sim p) \rightarrow q$ is contradiction, tautology or contingency.	**03**
	(b)	Show that the proposition $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ is a tautology.	**04**
	(c)	Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 18, 24} be ordered by the relation x divides y. Show that the relation is partial ordering and draw the Hasse diagram.	**07**
		OR	
	(a)	Show that the propositions $p \rightarrow q$ and $\sim p \vee q$ are logically equivalent.	**03**
	(b)	Negate (opposite) each of the following statements (i) $\forall x, \lvert x \rvert = x$ (ii) $\exists x, x^2 = x$ (iii) If there is a riot, then someone killed. (iv) It is day light and all the people are arisen.	**04**

(c) Solve the following recurrence relation **07**

(i) $a_n - 7a_{n-1} + 10a_{n-2} = 0$ given that $a_0 = 0$, $a_1 = 3$.

(ii) $a_n - 4a_{n-1} + 4a_{n-2} = 0$ given that $a_0 = 1$, $a_1 = 6$.

Q.4 (a) Explain Partially order relation, Poset and Hasse diagram with example(s) **03**

(b) Explain the regular graph and complete bipartite graph with example(s). **04**

(c) Define tree and root. Also draw a tree with 10 vertices which has vertices either of degree 1 or of degree 3. Is it possible to draw a same type of tree with 11 vertices? **07**

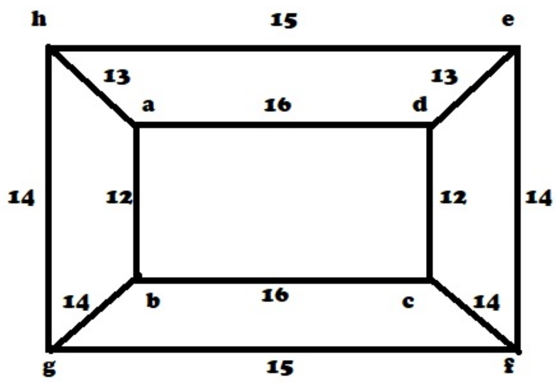
OR

(a) Let n be a positive integer, S_n be the set of all divisors of n. Let D denote the relation of division. Draw the lattices for (i) n = 24, (ii) n = 30 **03**

(b) Draw the following grapghs **04**

(i) $K_{3,4}$ (ii) $K_{4,4}$ (iii) $K_{5,4}$ (iv) K_5

(c) Find minimum spanning tree for the weighted graph given below **07**



Q.5 (a) Define abelian group for any non empty set. **03**

(b) If G = {0, 1, 2, 3, 4, 5, 6, 7} and operation $+_8$ is an addition modulo 8, then show that (G, $+_8$) is an abelian group. **04**

(c) Consider the set (**C, +, .**), where **C** is the set of complex numbers and + and . are ordinary addition and multiplication operation. Show that (**C, +, .**) is a field. **07**

OR

(a) Let Q_+ be the set of all positive rational numbers and ∗ be a binary operation on Q_+ defined by a∗b = ab/3 , then show that (Q_+, ∗) is an abelian group. **03**

(b) Define cyclic group and show that the following groups are cyclic groups **04**

(i) Third roots of unity i.e. G = {1, ω, ω^2}, $\omega^3 = 1$

(ii) G = {1, -1, i, -i}, where $i^2 = -1$

(c) Show that R = {a+b√2; a, b ∈ I} is an integral domain but not a field under the binary operations + (usual addition) and . (usual multiplication). **07**

***