Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–IV (NEW) EXAMINATION – WINTER 2024

Subject Code:2141905 **Date:19-11-2024**

Subject Name: Complex Variables and Numerical Methods

Time:02:30 PM TO 05:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

MARKS

Q.1 **(a)** Find the real and imaginary part of $f(z) = z^2 + 3z$ **03**

(b) Determine and sketch the regions in the z-plane represented by $1 < |z + 2i| \le 3$ **04**

(c) Define Analytic function. Check whether $f(z) = sinz$ is analytic or not. If analytic, find its derivative. **07**

Q.2 **(a)** Is $f(z) = \frac{z}{|z|}$ $(z \ne 0)$ is continuous at origin? **03**

= 0 for z = 0

(b) Find the principal value of $\left[\frac{e}{2}(-1 - i\sqrt{3})\right]^{3\pi i}$ **04**

(c) Verify Cauchy's integral theorem for $f(z) = z^2$ taken over the boundary of a square with vertices at $\pm 1 \pm i$ in counter-clockwise direction. **07**

OR

(c) Using the residue theorem, evaluate $\int_0^{2\pi} \frac{4d\theta}{5+4sin\theta}$ **07**

Q.3 **(a)** Define Harmonic function. Show that $u(x,y) = x^2 - y^2$is harmonic. **03**

(b) Evaluate $\oint_C \frac{dz}{z^2+1}$ where C is $|z + i| = 1$, counterclockwise. **04**

(c) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series valid for **07**

(i) $|z| < 1$ (ii) $1 < |z| < 3$

OR

Q.3 **(a)** Evaluate $\int_{-\infty}^{\infty} \frac{3z+2}{z(z-4)(z^2+9)} dz$ **03**

(b) Find $\oint_c tanz\, dz$, where C is the circle $|z| = 2$. **04**

(c) Find the bilinear transformation that maps the point $0,1,i$ in z-plane onto the points $1 + i, -i, 2 - i$ in the w-plane. **07**

Q.4 **(a)** Evaluate $\int_0^1 \frac{dx}{1+x^2}$, using trapezoidal rule with h=0.2 **03**

(b) Find a root of the equation $x^3 - x - 11 = 0$ using the bisection method up to fourth approximation. **04**

(c) Determine $y(12)$ by Lagrange Interpolation from the following **07**

X	11	13	14	18	20	23
Y	25	47	68	82	102	124

OR

Q.4 **(a)** Find the square root of 10 correct to three decimal places, by using Newton-Raphson iteration formula. **03**

(b) State Simpson's 3/8 rule and evaluate $\int_0^1 \frac{dx}{1+x^2}$ taking $h = \frac{1}{6}$ **04**

(c) Interpolate by means of Gauss's backward interpolation formula, the sales of a concern for the year 1966, given that **07**

Year	1931	1941	1951	1961	1971	1981
Sales(in Lakh Rs.)	12	15	20	27	39	52

Q.5 **(a)** Use Euler's method to solve the initial value problem $\frac{dy}{dx} = \frac{x-y}{2}$ on [0,3] with y(0)=1. **03**

(b) Use power method to find the largest eigen value of the matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$. Perform four iterations only. **04**

(c) Solve the following equations by Gauss-Seidel method. **07**

$$27x + 6y - z = 85$$
$$6x + 5y + 2z = 72$$
$$x + y + 54z = 110$$

OR

Q.5 **(a)** Using Taylor's series method, find $y(0.1)$ correct up to four decimal places given that $\frac{dy}{dx} = y - \frac{2x}{y},\ y(0) = 1$. **03**

(b) Use Runge – kutta second order method to find the approximate value of $y(0.2)$ given that $\frac{dy}{dx} = x - y^2$ and $(0) = 1$ and $h = 0.1$. **04**

(c) Solve $\frac{dy}{dx} = x + y$ with $y(0) = 1$ by Euler's modified method for $x = 0.1$ correct to four decimal places by taking $h = 0.05$. **07**

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