Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

M.SC (CS)- INTEGRATED– SEMESTER III- EXAMINATION –WINTER-2023

Subject Code:1330305 **Date: 06/12/2023**

Subject Name: Numerical Methods

Time: 10:30 AM TO 01:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make Suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Use of simple calculators and non-programmable scientific calculators are permitted.**

Q.1 **(a)** State Budan's Theorem. **03**

(b) Define Error. Explain different types of numerical errors with suitable example. **04**

(c) Derive an expression for Newton's forward difference interpolation formula. **07**

Q.2 **(a)** Define the following terms: Absolute Error, Relative Error, and Blunders. **03**

(b) Use Bisection method to find the root of the equation $x^3 - 5x + 1 = 0$, in the interval [2, 3], correct upto three decimal places. **04**

(c) Give diagrammatic representation of the Successive Approximation method to find the root of the equation f(x) = 0, for the cases of convergence as well as divergence. **07**

OR

(c) Solve the equation $x^4 - x - 10 = 10$ by Newton Raphson method, taking initial guess as 2.0. **07**

Q.3 **(a)** Find y(10) from the data given below using Lagrange's interpolation. **03**

x	5	6	9	11
y	12	13	14	16

(b) From the following data, find the value of y at x = 0.5, using Lagrange's interpolation formula. **04**

x	−2	−1	2	3
y	−12	−8	3	5

(c) Find y(46) and y(63) from the below given data using Newton's interpolation: **07**

Age (x)	45	50	55	60	65
Premium (y)	114.84	96.16	83.22	74.48	68.48

OR

Q.3 **(a)** Obtain Cubic Spline equation for subinterval [0, 1] for the data given in the table: **03**

x	0	1	2	3
f(x)	1	2	33	244

(b) Fit a second degree parabola of the form $y = ax^2 + bx + c$ to the following data by using method of least squares: **04**

x	1	2	3	4	5
y	5	12	26	60	97

(c) Determine the curve of the form $\mathbf{y = a.x^b}$, which is the best fit to the following data according to least square equation. **07**

x	1.0	1.5	2.0	2.5	3.0	3.5
y	0.01	0.405	0.693	0.916	1.098	1.252

Q.4 **(a)** From the data, find numerically the first and second order derivatives at x = 1.3. **03**

x	0.5	0.7	0.9	1.1	1.3	1.5

y	1.48	1.64	1.78	1.89	1.96	2.00

(b) The table below gives the results of an observation, 'θ' is the observed temperature in degrees centigrade of a vessel of cooling water, 't' is the time in minutes from the beginning of observation. Find the appropriate rate of cooling at t = 3 and t = 3.5. **04**

t	1	3	5	7	9
θ	85.3	74.5	67.0	60.5	54.3

(c) Find the first two derivatives of '$x^{1/3}$' at x = 50 and x = 56 from the table below: **07**

x	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

OR

Q.4 **(a)** From the data, find numerically the first and second order derivatives at x = 1.1. **03**

x	1.0	1.2	1.4	1.6	1.8	2.0
y	0	0.128	0.544	1.296	2.432	4.000

(b) A Curve passes through the points (1, 2), (1.5, 2.4), (2.0, 2.7), (2.5, 2.8), (3, 3), (3.5, 2.6) and (4.0, 2.1). Obtain the area bounded by the curve, the X-axis and x = 1 and x = 4. **04**

(c) A river is 80 metres wide. The depth 'd' in metres at a distance 'x' metres from one bank is given by the following table. Calculate the area of cross-section of the river using Simpson's 1/3 rule. **07**

x (distance in metres)	0	10	20	30	40	50	60	70	80
d (depth in metres)	0	4	7	9	12	15	14	8	3

Q.5 **(a)** Use Milne-Simpson's Predictor corrector formula to solve **03**
$y' = 2y - y^2$, for x = 0.2 and x = 0.25 if
y(0) = 1
y(0.05) = 1.0499584
y(0.10) = 1.0996680
y(0.15) = 1.148850

(b) Given the following differential equation $y' = x^2 + x^4 y$, with y(0) = 3 and starting values y(0.1) = 3.0050, y(0.2) = 3.0202 and y(0.3) = 3.0465. **04**
Find y(0.4) using Adam–Bashforth–Moulton's Predictor–Corrector method.

(c) Solve the following system of linear equations using Gauss-Seidel method: **07**
$2x_1 - 2x_2 + 5x_3 = 13$
$2x_1 + 3x_2 + 4x_3 = 20$
$3x_1 - x_2 + 3x_3 = 10$

OR

Q.5 **(a)** Solve the following ordinary differential equation using Taylor series method: **03**
$y' = y^2 + x$; given that y(0) = 0, find y(0.2).

(b) Solve the following differential equation y' = x – 2y, y(0) = 1, using Runge–Kutta 4th order method to find y(0.1) and y(0.2). **04**

(c) Use Runge Kutta 4th order method to solve y(0.2) and y(0.4) when **07**
$y' = (2xy + e^x) / (x^2 + x.e^x)$ given that y(0) = 0 and h = 0.2.

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