GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER–VII (NEW) EXAMINATION – WINTER 2023

Subject Code:3171611   **Date:08-12-2023**

Subject Name: Graph Theory and Combinatorics
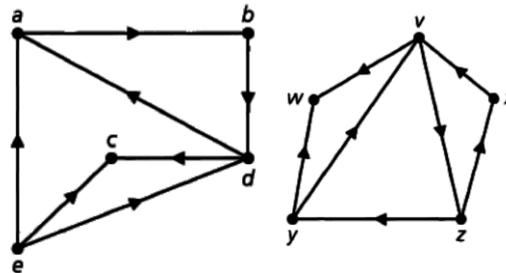
Time: 10:30 AM TO 01:00 PM   **Total Marks:70**

Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 (a) Check given two graphs are isomorphic or not. Justify your answer.   **03**


(b) Prove that the number of vertices of odd degree in a graph is always even. Also, take any graph and verify it.   **04**

(c)   1) Define Spanning Tree.   **07**
   2) In any tree (with two or more vertices), there are at least _____ pendant vertices.
   3) If a given tree be a complete 3-ary tree with 25 leaves then what is the height of a tree?
   4) Let L_1 and L_2 be two sorted lists of ascending numbers, where L_1 contains 10 elements and L_2 contains 6 elements. Then L_1 and L_2 can be merged into one ascending list L than at most how many comparisons are required.
   5) Does the given graph a tree? G=(V,E) with V={a,b,c,d,e} and E={{a,b},{a,e},{b,c},{c,d},{d,e}}.
   6) A code for {a, b, c, d, e} is given by a:00, b:01, c:101, d:x10, e:yz1, where x, y, z $\in$ {0, 1}. Determine x, y and z so that the given code is a prefix code.
   7) Let T = (V, E) be a binary tree. In Fig, we find the subtree of T rooted at vertex p. (The dashed line coming into vertex p indicates that there is more to the tree T than what appears in the figure.) If the level number for vertex u is 37, what are the level numbers for vertices p, s , t , x, y, and z?
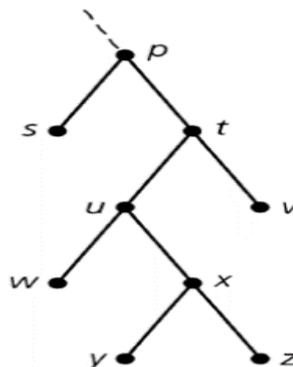
Q.2 (a) For the network shown in fig. let the capacity of each edge be 10. If each edge e in the figure is labeled by a function f, as shown, determine the values of s, t, w, x and y so that f is a flow in the network. Also find the value of this flow. **03**
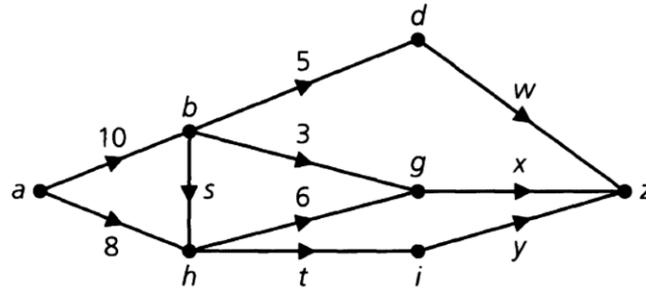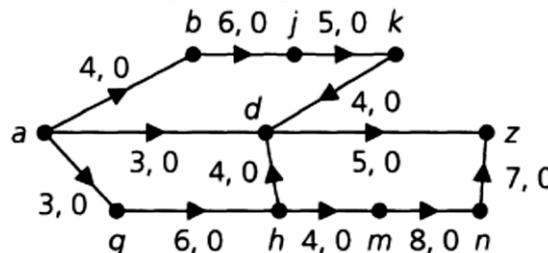

(b) Find Dual Graph For the given Graph G. **04**
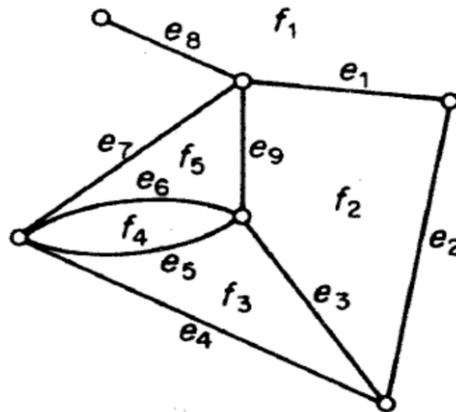
(c) 1) Describe Königsberg bridge problem. **07**
2) Following graph Contains a Hamilton path or Hamilton Cycle? Justify your answer.
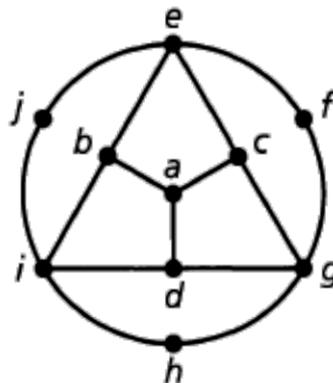
OR

(c) Find Max flow min cut value for a given graph. **07**

Q.3 (a) 1) In a graph the total number of edges are 87 than the sum of degrees of all vertices is **03**
_____.

2) What is the chromatic Number for Petersen graph ?

3) If Graph G(V,E) is a Complete Graphs then Chromatic Polynomials $P(G, \lambda) =$ _____.

(b) Five Faculties Rajesh, Divy, Shivam, Brijesh and Paresh are members of three sports games, Cricket, Football and wallyball. Each game is to select a Faculty representative for **04**

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manage game properly. Can a selection be made in such a way that each game has a distinct representative? For solving this problem construct a transport network.
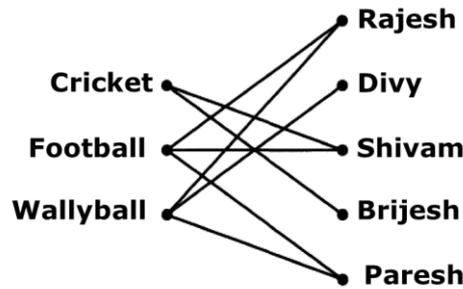
(c) Determine which of the graph in give figure are planar. If a graph is planar, redraw it with no edges overlapping. If it is non planar, find a subgraph homeomorphic to either K_5 or $K_{3,3}$. **07**

(a)   (b)   (c)
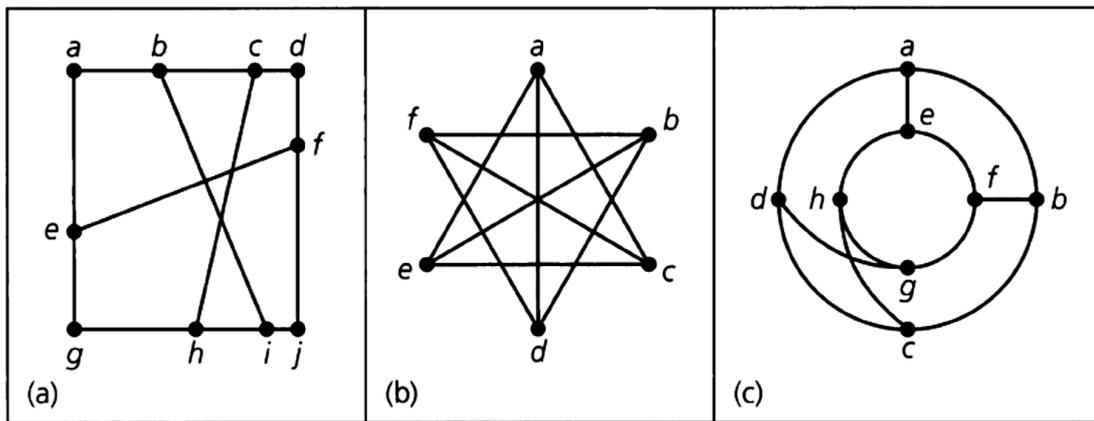
OR

Q.3 **(a)** A classroom contains 25 microcomputers that must be connected to a wall socket that has four outlets. Connections are made by using extension cords that have four outlets each. What is the least number of cords needed to get these computers set up for class use? **03**

(b) For given preference of men and women find the solution for stable marriage problem. **04**

Men

A	O	M	N	L	P
B	P	N	M	L	O
C	M	P	L	O	N
D	P	M	O	N	L
E	O	L	M	N	P

Women

L	D	B	E	C	A
M	B	A	D	C	E
N	A	C	E	D	B
O	D	A	C	B	E
P	B	E	A	C	D

(c) Using proper method to find at least 3 De Bruijn Sequence $B(k,n)$ for Input $n = 2$, $k = 3$ (Base). **07**

Q.4 **(a)** Ms Pezzulo teaches geometry and then biology to a class of 12 advanced students in a classroom that has only 12 desks. In how many ways can she assign the students to these desks so that **03**
(a) no student is seated at the same desk for both classes?
(b) there are exactly six students each of whom occupies the same desk for both classes?

(b) Determine the number of positive integers n, $1 \leq n \leq 2000$ that are **04**
1) not divisible by 2, 3 or 5
2) not divisible by 2, 3, 5 or 7
3) not divisible by 2, 3 or 5 but are divisible by 7

(c) 1) The MASSASAUGA is a brown and white venomous snake indigenous to North America. Arranging all of the letters in MASSASAUGA, we find that there are how many possible arrangements. **07**

2) A student taking a history examination is directed to answer the seven of 10 essay questions. If the student must answer three questions from the first five and four questions from the last five, so How many ways the student can answer the examination?

3) A message is made up of 15 different symbols and is to be transmitted through a communication channel. In addition to the 15 symbols, the transmitter will also send a total of 60 (blank) spaces between the symbols, with at least three spaces between each pair of consecutive symbols. In how many ways can the transmitter send such a message?

OR

Q.4 **(a)** While on a Saturday shopping spree Jennifer and Tiffany witnessed two men driving away from the front of a jewellery shop, just before a burglar alarm started to sound. Although everything happened rather quickly, when the two young ladies were questioned, they were able to give the police the following information about the license plate (which consisted of two letters followed by four digits) on the get-away car. Tiffany was sure that the second letter on the plate was either an O or a Q and the last digit was either a 3 or an 8. Jennifer told the investigator that the first letter on the plate was either a C or a G and that the first digit was definitely a 7. How many different license plates will the police have to check out? **03**

(b) In how many ways can one travel in the x y-plane from (0, 0) to (5, 5) using the moves R:(x, y) → (x + 1, y) and U: (x, y) → (x , y + 1), if the path taken may touch but never fall below the line y = x? **04**

(c) In how many ways can one arrange the letters in CORRESPONDENTS so that (a) there is no pair of consecutive identical letters? (b) there are exactly two pairs of consecutive identical letters? (c) there are at least three pairs of consecutive identical letters? **07**

Q.5 **(a)** Determine the generating function for the sequence $a_o, a_1, a_2....$, where a_n is the number of partitions of the nonnegative integer n into (a) even summands; (b) distinct even summands; (c) distinct odd summands. **03**

(b) In how many ways can two dozen identical robots be assigned to four assembly lines with (a) at least three robots assigned to each line? (b) at least three, but no more than nine, robots assigned to each line? **04**

(c) Define Homogeneous and non-Homogeneous Recurrence Relation. Find the unique solution of the recurrence relation $a_n - 3a_{n-1} = 5(7^n)$, where n ≥ 1 and $a_0 = 2$. **07**

OR

Q.5 **(a)** The number of bacteria in a culture is 1000 (approximately), and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. **03**

(b) Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$, where n ≥ 2 and $a_0 = -1, a_1 = 8$ **04**

(c) "In how many ways can four of the letters in ENGINE be arranged?" Solve using exponential generating function. **07**

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