Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – WINTER 2023

Subject Code:2141905 Date:11-01-2024

Subject Name:Complex Variables and Numerical Methods

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** If Z_1 and Z_2 be two complex numbers, show that $\left(Z_1+Z_2\right)^2+\left(Z_1-Z_2\right)^2=2\left[\left|Z_1\right|^2+\left|Z_1\right|^2\right]$ **03**

(b) Simplify $\dfrac{\left(\cos 2\theta+i\sin 2\theta\right)^{\frac{2}{3}}\left(\cos\theta-i\sin\theta\right)^2}{\left(\cos 3\theta-i\sin 3\theta\right)^2\left(\cos 5\theta-i\sin 5\theta\right)^{\frac{1}{3}}}$ **04**

(c) Evaluate $\left(1+i\sqrt{3}\right)^{90}+\left(1-i\sqrt{3}\right)^{90}$ **07**

Q.2 **(a)** If $\arg\left(z+1\right)=\dfrac{\pi}{6}$ and $\arg\left(z-1\right)=\dfrac{\pi}{3}$, then find the complex number Z. **03**

(b) If $1+2i$ is a root of the equation $Z^4-3Z^3+8Z^2-7Z+5=0$, then find all other roots. **04**

(c) Solve $z^4+1=0$ and locate the roots in argand diagram. **07**

Q.3 **(a)** Find the real and imaginary part of $\dfrac{5+2i}{1-i}$ **03**

(b) Define Harmonic function and determine whether the function $\phi\left(x,y\right)=e^x\cos y$ is harmonic? **04**

(c) Using the residue theorem, evaluate $\int_0^{2\pi}\dfrac{d\theta}{5-3\sin\theta}$ **07**

OR

Q.3 **(a)** Determine the analytics function whose real part is $u\left(x,y\right)=x^2-y^2$.

(b) Find the bilinear transformation that maps respectively the points $i,1,-i$ in z - plane onto the points $-i,1,i$ in w – plane.

(c) Evaluate $\int_{-\infty}^{\infty}\dfrac{dx}{\left(x+1\right)\left(x^2+3\right)}$

Q.4 **(a)** Find the real root of the equation $x^3+4x^2-1=0$ by bisection method. **03**

(b) Evaluate $\int_0^6 \frac{1}{1+x}dx$ taking h = 1 using Simpson's 1/3 rule. **04**

(c) Compute the value of f(7.5), by using suitable interpolation formula using the following table of data. **07**

x	3	4	5	6	7	8
F(x)	28	65	126	217	344	513

OR

Q.4 **(a)** Find the positive root of $x = \cos x$ using Newton's method correct to 3 decimal places. **03**

(b) State the trapezoidal rule with n = 10 and evaluate $\int_0^1 e^x dx$ **04**

(c) Using Lagrange's interpolation formula, find the value of f (0) for the table given below : **07**

x	-1	-2	2	4
F(x)	-1	-9	11	69

Q.5 **(a)** Using Euler's method, find $y(0.2)$ given $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1$, take h = 0.1 **03**

(b) Solve the following system of equations by Gauss Seidel method. **04**
$10x + y + z = 6, x + 10y + z = 6, x + y + 10z = 6$

(c) Use the Runge-Kutta method to solve $\frac{dy}{dx} = -xy^2 \; for \; 0 \le x \le 1$, subject to $y(0) = 2$. Use h = 0.25 and find $y(0.5)$. **07**

OR

Q.5 **(a)** Using Taylor's series method, find correct to four decimal places, the value of $y(0.1)$, given $y' = x^2 + y^2$ and $y(0) = 1$. **03**

(b) Solve the following system of equations by Gauss Seidel method. **04**
$20x + 2y + z = 30, x - 40y + 3z = -75, 2x - y + 10z = 30$

(c) Use the Runge-Kutta method to solve $\frac{dy}{dx} = x + y$, subject to $y(0) = 0$. Use h = 0.2 and find $y(0.4)$. **07**

****