Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I (NEW) EXAMINATION – WINTER 2023

Subject Code:2110015 Date:23-01-2024

Subject Name:Vector Calculus And Linear Algebra

Time:02:30 PM TO 05:30 PM Total Marks:70

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 (a) In each of the following choose the correct option. (MCQ) **Marks 07**

1. Rank of the matrix $A = \begin{bmatrix} 1 & 1 & -3 \\ 3 & -1 & -1 \\ 2 & 1 & -2 \end{bmatrix}$ is
(a) 3 (b) 1 (c) -1 (d) 2
2. IF A is an invertible matrix then $A^{-1} =$ ______ is
(a) A (b) $\frac{adj\,A}{|A|}$ (c) adj A (d) adj A $|A|$
3. For what values of k does the system x+ky=1, kx+y=1 have no solution
(a) 1 (b) 2 (c) -1 (d) -2
4. The mapping $T: R^2 \to R^2$ defined by T(x , y) = (x , 0) is called as
(a) Rotation (b) Projection (c) Reflection (d) Contraction
5. The matrix $\begin{bmatrix} 1 & 3 & -3 \\ 0 & 2 & -1 \\ 0 & 0 & -2 \end{bmatrix}$ is ......
(a) upper triangular (b) lower traiangular (c) diagonal (d) none of these
6. The matrix $\begin{bmatrix} 0 & -5 & -10 \\ 5 & 0 & 2 \\ 10 & -2 & 0 \end{bmatrix}$ is ......
(a) symmetric (b) diagonal (c) triangular (d) skew symmetric
7. The mapping $T: R^3 \to R^3$ defined by T(X) = X is called as
(a) Identity (b) Projection (c) Reflection (d) Contraction

(b) In each of the following choose the correct option. (MCQ) **07**

1. The divergence of the vector $v = (2x - y^2, 3z + x^2, 4y - z^2)$ at the point (1,2,3) is .....
(a) 0 (b) 2 (c) 1 (d) -4
2. Find the values of k for which u=(k,k,1) and v=(k,5,6) are orthogonal.
(a) 2,3 (b) -2,-3 (c) 1,0 (d) -1,0
3. Find the vector v in R^3 whose coordinate vector with respect to the basis s = {(1,1,0),(1,0,1),(0,1,1)} is $v_s = (1, -2, 3)$
(a) (-1,4,1) (b) (2,2,1) (c) (1,1,1) (d) None of these
4. The angle between the vectors u = (-2 , 3 , 4) and v = (2 , 4, -2) is
(a) 60° (b) 30° (c) 15° (d) 90°
5. The vectors $v_1 = (2,3)$ and $v_2 = (3, -1)$, $v_3 = (5,2)$ are
(a) Linearly dependent (b) Linearly independent (c) Both (a) and (b) (d) none of these
6. If τ_1 and τ_2 are eigen values of matrix A then $\frac{1}{\tau_1}, \frac{1}{\tau_2}$ are eigen values of
(a) 2A (b) A^{-1} (c) $(A^{-1})^{-1}$ (d) none of these
7. If A=$\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ then the eigen values of $4A^{-1}$ are
(a) 1,2 (b) 2,3 (c) 4,4 (d) 4,1

Q.2	(a)	If $u = (1,2,2)$ and $v = (3,4,6)$ then prove that w=(5,8,10) is a linear combination of u and v.	03
	(b)	Find the inverse of the following matrix by Gauss-Jordan method. $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$	04
	(c)	Let R^3 have Euclidean inner product. Use the gram-schmidt process to transform the basis vectors $u_1 = (1,1,1), u_2 = (0,1,1), u_3 = (0,0,1)$ into orthonormal basis.	07
Q.3	(a)	Determine whether the following vector span the vector space R^3 $v_1 = (1,0,1), v_2 = (2,1,4), v_3 = (1,1,2)$.	03
	(b)	Test for consistency and solve $x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6$	04
	(c)	If s={(1,1,1), (1,-1,1), (0,1,1)} is basis for the vector space R^3 then (1) Find the coordinate vector of v=(1,2,3) with respect to s (2) Find the vector v in R^3whose coordinate vector with respect to s is (2,4,-6).	07
Q.4	(a)	Show that the function $T: R^2 \to R^2$ defined by T(x,y)=(x+2y,3x-y) is a linear transformation.	03
	(b)	Find $(T_2 \circ T_1)(x, y)$ for the transformation given by $T_1(x, y) = (x - 2y, 0), T_2(x, y) = (2x - 5y, 3x + 4y)$	04
	(c)	Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$	07
Q.5	(a)	Determine the constant b such that $\bar{F} = (bx + 4y^2z)\hat{\imath} + (x^3sinz - 3y)\hat{\jmath} - (e^x + 4cosx^2y)\hat{k}$ is solenoidal.	03
	(b)	If R^3 has the Euclidean inner product then find k such that u and v are orthogonal. (1) U=(k,k,1), v=(k,5,6) (2) u=(2,1,3) , v=(1,7,k)	04
	(c)	Find the characteristic equation of the matrix A and verify that it satisfies Cayley-Hamilton theorem. Where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$	07
Q.6	(a)	Evaluate $\int_C f(x, y, z)ds$ over the line segment C joining the point (0.1.0) to the point (1,0,0) where $f(x, y, z) = x + y$	03
	(b)	Using the inner product $\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)dx$ in $C(-1,1)$ $find$ $\langle f, g\rangle$ where f(x)=2+x, g(x) = $1 + x^2$	04
	(c)	Use Green's theorem to evaluate the integral $\oint_C (x^2 - 2xy)dx + (x^2y + 3)dy)$ where C is the boundry of the region bounded by the parabola $y = x^2$ and the $line$ $y = x$.	07
Q.7	(a)	Find the directional derivative of $xy^2 + z^2y$ at (2,-1,1) in the direction of i +2 j+ 2k.	03
	(b)	Find the scalar potential function $\emptyset$ for $\bar{F} = (x^2 + xy^2)\hat{\imath} + (y^2 + x^2y)\hat{\jmath}$	04
	(c)	Find a matrix P which diagonalises the matrix $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$	07

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