Seat No.:_____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M.Sc. INTEGRATED (COMPUTER SCIENCE) - SEMESTER III
EXAMINATION- WINTER-2022

Subject Code: 1330305 Date: 28-12-2022

Subject Name: Numerical Methods

Time: 10:30 AM TO 01:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** Construct the difference table for $f(x) = (x+2)^2, x = 1,2,3$ and find $\nabla^2 f(3)$. **03**

(b) Write the Iterative formula of NR method. Using it find the first iteration of the solution of $x^3 - x - 1 = 0$. **04**

(c) Find the approximate root of the equation $x - \cos x = 0$ up to three decimal places using bisection method. **07**

Q.2 **(a)** Prove that $(1+\Delta)(1-\nabla) = 1$. **03**

(b) Using trapezoidal rule evaluate $\int_0^1 \frac{1}{1+x} dx$ for $h = \frac{1}{4}$. **04**

(c) Find the polynomial using Newton's forward interpolation formula from the following data: **07**

x	1	2	3	4
f(x)	-1	-1	1	5

OR

(c) Find $\Gamma(1.01)$, using Newton's backward interpolation formula from the following: **07**

x	1	1.02	1.04
$\Gamma(x)$	1.0000	0.9888	0.9784

Q.3 **(a)** Using inverse Lagrange's interpolation formula find x for $y = 5$ from the following table. **03**

x	1	3	4
y	3	12	19

(b) If $f(x) = {}^{1}/_{x}$, find the divided differences $[a, b]$ and $[a, b, c]$. **04**

(c) Construct a second degree polynomial using Lagrange's interpolation formula from the following **07**

x	0.3	0.5	0.7
f(x)	0.61	0.69	0.72

Hence Compute $f(0.4)$

OR

Q.3 **(a)** Using Simpson's $\frac{1}{3}$rd rule evaluate $\int_1^{1.8} e^{-x} dx$ for $h = 0.2$. **03**

(b) Using Simpson's $\frac{3}{8}$th rule evaluate $\int_0^6 e^x dx$ for $h = 1$. **04**

(c) Using Newton's divided difference formula, compute $f(10.5)$ from the following: **07**

x	10	11	13	17
f(x)	2.3026	2.3979	2.5649	2.8332

Q.4 **(a)** Applying Budan's theorem, calculate the number of real roots of $x^5 - x^3 + 1 = 0$. **03**

(b) Apply Runge-Kutta method of order two, calculate $y(1.1)$, given that $\frac{dy}{dx} = x - y$ with $y(1) = 0$ taking $h = 0.1$. **04**

(c) Apply Runge-Kutta method of forth order to calculate $y(0.2)$, given that $\frac{dy}{dx} = x + y$ with $y(0) = 1$ taking $h = 0.1$. **07**

OR

Q.4 **(a)** Using Euler's method, find $y(0.02)$ from the initial value problem $y' = y, y(0) = 1$ by taking step size $h = 0.01$. **03**

(b) Find $y(0.2)$ from $\frac{dy}{dx} = x + y$ with $y(0) = -1$ using Taylor's series method. **04**

(c) Using Gauss Jacobi method, solve the following system up to three decimal places. **07**

10x+y+z=6, x+10y+z=6 , x+y+10z=6

Q.5 **(a)** Fit a straight line using least square method from the following data: **03**

x	0	1	2
y	1	3	5

(b) Using modified Euler's method, find $y(0.1)$ from the initial value problem $\frac{dy}{dx} = xy, y(0) = 1$ by taking step size $h = 0.1$. **04**

(c) Using Gauss Seidel method, solve the following system up to three decimal places. **07**

5x+y+2z=19, x+4y-2z=-2 , 2x+3y+8z=39

OR

Q.5 **(a)** Write down the condition for convergence of the system of three linear equations. **03**

(b) Write down the normal equations for the fitting of linear curve using least square method. **04**

(c) Fit a second degree polynomial using least square method from the following data: **07**

x	0	1	-1
y	12	9	4

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