Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I & II (NEW) EXAMINATION – WINTER 2022

Subject Code:2110015 Date:14-03-2023

Subject Name:Vector Calculus And Linear Algebra

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Marks

Q.1 (a) Objective Questions (MCQ) **07**

1. If A is invertible $n \times n$ matrix, then for each $n \times 1$ matrix b, the system of equations $Ax = b$ has _____solution.
   (a) Infinitely many (b) no solution (c) exactly one (d) none of these
2. If u and v are vectors in R^n then the Cauchy-Schwarz inequality is__
   (a)$|u.v| \leq \|u\|\|v\|$ (b) $|u.v| \geq \|u\|\|v\|$
   (c) $|u.v| \neq \|u\|\|v\|$ (d) none of these
3. The eigen values of the matrix $\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ are
   (a) $2, -1$ (b) $0, 2$ (c) $1, 0$ (d) $1, -1$
4. The set $s = \{(0,1,0), (1,0,1), (1,0,-1)\}$ is an orthogonal set of vectors in an inner product space then the set s is _____
   (a) linearly independent (b) linearly dependent
   (c) both (d) none of these
5. If the matrix A of order n is orthogonally diagonalizable then A is
   (a) symmetric (b) skew symmetric (c) unitary (d) Hermitian
6. The vector $\bar{V} = (x + 3y)\hat{\imath} + (y - 2z)\hat{\jmath} + (x + \lambda z)\hat{k}$ is solenoidal for the value of $\lambda =$ ___
   (a) 2 (b) 0 (c) -2 (d) -1
7. If $\phi = x + y + z$ then the value of $|grad\ \phi|$ is
   (a) $\sqrt{3}$ (b) 0 (c)3 (d) 2

(b) Objective Questions (MCQ) **07**

1. The rank of the matrix whose every element is unity, is ___________
   (a) Zero (b) Greater than one (c) 2 (d) Equals to one
2. The set $\{(1,0), (1,1)\}$ is____
   (a) linearly dependent (b) linearly independent
   (c) Basis of R^2 (d) none of these
3. If $2, 3, 4$ are eigen values of the matrix A, then the eigen values of A^T are________
   (a) $1/2, 1/3, 1/4$ (b) $2, 3, 4$ (c) $4, 9, 16$ (d) Zero
4. $T: V \to V$ is a linear operator and V is a finite dimensional vector space, then T is one-to-one if
   (a) $R(T) = V$ (b) $nullity(T) \neq 0$ (c) $ker(T)\ ) \neq 0$ (d) none of these
5. The vectors $u = (2, k, 6)$ and $v = (1, 2, 3)$ are orthogonal with respect to Euclidean inner product, then the value of k $=$ ____
   (a) 10 (b) -5 (c) $zero$ (d) -10
6. If $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ the $\nabla.\bar{r} =$ ___
   (a) 0 (b) 3 (c) 1 (d) -3

7. The field F is conservative on D then the value of $\int F \cdot dr$ around every closed loop in D is____
(a) zero (b) -1 (c) none of these (d) 1

Q.2 **(a)** Find the coordinate vector of $(2,-1,3)$ relative to the basis $s = \{v_1, v_2, v_3\}$, where $v_1 = (1,0,0)$, $v_2 = (2,2,0)$ and $v_3 = (3,3,3)$ **03**

(b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ by using Gauss Jordan elimination method. **04**

(c) Investigate for which values of a, the system of equations $x_1 + x_2 + x_3 = 1, x_1 + 2x_2 + 4x_3 = a, x_1 + 4x_2 + 10x_3 = a^2$ has no solution, unique solution or infinitely many solutions. **07**

Q.3 **(a)** Find the kernel of the linear operator $T: R^2 \to R^2$ defined by $T(x,y) = (x-y,\ x-y)$ and determine whether the linear transformation T is one-to-one. **03**

(b) Consider the bases $S = \{v_1, v_2\}$ for R^2, where $v_1 = (-2,1)$ and $v_2 = (1,3)$ and let $T: R^2 \to R^3$ be the linear transformation such that $T(v_1) = (-1,2,0)$ and $T(v_2) = (0,-3,5)$. Find a formula for $T(x_1, x_2)$ and use that formula to find $T(2,-3)$. **04**

(c) Determine whether the set of all pairs of real numbers (x,y) with the operations $(x,y) + (x',y') = (x+x', y+y')$ and $k(x,y) = (2kx, 2ky)$ is a vector space. **07**

Q.4 **(a)** Show that the set of functions $\{1, e^x, e^{2x}\}$ form a linearly independent set of vectors in $C^2(-\infty, \infty)$. **03**

(b) Determine whether the vector $(7,8,9)$ is a linear combination of vectors $u = (2,1,4), v = (1,-1,3)$ and $w = (3,2,5)$. **04**

(c) Find a matrix P that diagonalizes $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ and determine $P^{-1}AP$. **07**

Q.5 **(a)** Determine whether the transformation $T: R^2 \to R^2$ defined as $T(x,y) = (x+3y, 3x-y)$ is linear or not? **03**

(b) Find the least squares solution of the linear system $Ax = b$ given by $x_1 - x_2 = 4, 3x_1 + 2x_2 = 1, -2x_1 + 4x_2 = 3$ and find the orthogonal projection of b on the column space of A. **04**

(c) Let R^3 have the Euclidean inner product. Use Gram-Schmidt process to transform the basis $\{(1,1,1), (-1,1,0), (1,2,1)\}$ into an orthonormal basis. **07**

Q.6 **(a)** Find the line integral of $f(x,y) = \frac{x^3}{y}$ over the curve C: $y = \frac{x^2}{2}, 0 \le x \le 2$. **03**

(b) How much work is required to move an object in vector force field $F = \langle yz, xy, xz \rangle$ along path $r(t) = \langle t^2, t, t^4 \rangle, 0 \le t \le 1$? **04**

(c) Define curl and divergence of a vector field. The vector field is given by $\bar{A} = (x^2 + xy^2)\hat{\imath} + (y^2 + x^2y)\hat{\jmath}$. Show that the field $\bar{A}$ is irrotational and also find the scalar potential. **07**

Q.7 **(a)** Use the inner product $\langle f, g\rangle = \int_0^1 f(x)g(x)dx$; to compute $d(f, g)$ and $\|f\|$, where $f(x) = x,\ g(x) = e^x$ in $C[0,1]$. **03**

(b) Find the directional derivative of the function $\phi = xy^2 + yz^2$ at point $P(2, -1, 1)$ along the tangent to the curve $x = atsint,\ y = atcost,\ z = at$ at $t = {}^{\pi}/_{4}$. **04**

(c) Verify Green's theorem for $\oint_c [(xy + y^2)dx + x^2 dy]$ where C is bounded by $y = x$ and $y = x^2$ **07**

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