Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I & II(NEW) EXAMINATION – WINTER 2022

Subject Code:2110014 Date:02-03-2023

Subject Name:Calculus

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
1. **Make suitable assumptions wherever necessary.**
2. **Figures to the right indicate full marks.**
3. **Simple and non-programmable scientific calculators are allowed.**

			Mark
Q.1		**Objective Question (MCQ)**	
	(a)	Choose the appropriate answer for the following questions:	**07**
	1.	The Series $\sum_{n=1}^{\infty} \frac{n^2+4}{n^p}$ is convergent if (a) $p < 3$ (b) $p > 3$ (c) $p = 3$ (d) $p > -1$	
	2.	The series $1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \ \ldots$ is (a) Convergent (b) Divergent (c) Oscillates finitely (d) Oscillates infinitely	
	3.	The Maclaurin series expansion of $\sin x$ is (a) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$ (b) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$ (c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ (d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$	
	4.	The value of $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2+y^2}$ is (a) 1 (b) $1/2$ (c) 0 (d) 2	
	5.	$\int_0^1 \int_0^2 \int_0^3 dz\, dy\, dx =$ _______ (a) 12 (b) 3 (c) 0 (d) 6	
	6.	If $u = \frac{y}{x}$, $v = xy$ then $\frac{\partial(u,v)}{\partial(x,y)}$ =_________ (a) $-\frac{2y}{x}$ (b) $-\frac{2x}{y}$ (c) $\frac{2y}{x}$ (d) $\frac{2x}{y}$	
	7.	If $u = x^y$ then $\frac{\partial u}{\partial x}$ is (a) 0 (b) yx^y (c) $x^y \ln x$ (d) $y\, x^{y-1}$	
	(b)	Choose the appropriate answer for the following questions:	**07**
	1.	If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is (a) u (b) $-u$ (c) 0 (d) 1	
	2.	The value of $\lim_{x\to\infty} x\, e^{-x}$ is (a) 1 (b) 0 (c) e (d) $1/e$	

3. The value of $\lim_{x \to 0} \frac{\sin x}{x}$ is

(a) 1 (b) 0 (c) π (d) ∞

4. Which of the following curve passes through the origin?

(a) $xy^2 = 4a(3a - x)$ (b) $y^2(a - x) = x^2(x + 2)$

(c) $y^2(a - x) = x + 5$ (d) None of these

5. If in the equation of a curve, x occurs only as an even power then the curve is symmetric about___________.

(a) Origin (b) x-axis (c) y-axis (d) None

6. The curve $r = a(1 + \cos\theta), a > 0$ represents_______.

(a) Circle (b) Cardioid

(c) Lemniscate of Bernoulli (d) None of these

7. $\iint r^3\, dr d\theta$ over the region included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$ is

(a) $\int_0^{\pi}\int_{\sin\theta}^{2\sin\theta} r^3\, dr d\theta$ (b) $\int_0^{\frac{\pi}{2}}\int_{2\sin\theta}^{4\sin\theta} r^3\, dr d\theta$

(c) $\int_{-\pi}^{\pi}\int_{2\sin\theta}^{4\sin\theta} r^3\, dr d\theta$ (d) $\int_0^{\pi}\int_{2\sin\theta}^{4\sin\theta} r^3\, dr d\theta$

Q.2 **(a)** Test the convergence of the series: $\sum_{n=1}^{\infty} \frac{2n+3}{(n+1)^2}$ **03**

(b) Test the convergence of the series: $\sum_{n=1}^{\infty} \frac{2^n}{n^3+1}$ **04**

(c) Find the interval of convergence for the series, **07**

$$2x + \frac{3}{8}x^2 + \frac{4}{27}x^3 + \frac{5}{64}x^4 + \ ...$$

Q.3 **(a)** Evaluate: $\int_0^{\infty} \frac{1}{x^2 + 1}\, dx$ **03**

(b) Evaluate: $\lim_{x \to 0} \frac{5\sin x - 7\sin 2x + 3\sin 3x}{\tan x - x}$ **04**

(c) Trace the curve: $xy^2 = 4a^2(a - x)$ **07**

Q.4 **(a)** Expand $3x^3 + 8x^2 + x - 2$ in powers of $x - 3$. **03**

(b) Evaluate: $\int_0^{\pi}\int_0^{\sin\theta} r\, dr d\theta$ **04**

(c) Find the volume generated by revolving the area cut off from the parabola $9y = 4(9 - x^2)$ by the line $4x + 3y = 12$ about x −axis. **07**

Q.5 **(a)** If $u = e^{3xyz}$, show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (3 + 27xyz + 27x^2y^2z^2)e^{3xyz}$ **03**

(b) Find the equation of the tangent plane and normal line to the surface $x^2 + y^2 + z^2 = 1$ at (2,2,1). **04**

(c) (i) If $u = \frac{1}{3}\log\left(\frac{x^3+y^3}{x^2+y^2}\right)$, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ **03**

(ii) Expand e^{x+y} in power of $x - 1$ and $y + 1$ by using Taylor's series. **04**

Q.6 **(a)** If $u = \log(x^2 + y^2)$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. **03**

(b) If $z = e^{ax+by} f(ax - by)$, prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$ **04**

(c) Find the maximum and minimum distances of the point $(3,4,12)$ from the sphere $x^2 + y^2 + z^2 = 1$. **07**

Q.7 **(a)** Evaluate: $\int_0^2 \int_1^2 \int_0^{yz} xyz \, dx \, dy \, dz$ **03**

(b) Evaluate: $\iint_R (x + y)^2 dxdy$ **04**

Where R is the region bounded by $x + y = 0, x + y = 1, 2x - y = 0,$ $2x - y = 3$ using transformations $u = x + y$ and $v = 2x - y$.

(c) (i) Evaluate: $\iint_R r^3 \, dr d\theta;$ where R is the region bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$. **03**

(ii) By changing the order of integration, Evaluate the integral **04**

$$\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} dydx$$

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