GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER–IV (NEW) EXAMINATION – WINTER 2021
Subject Code:2141905   **Date:05/01/2022**
Subject Name:Complex Variables and Numerical Methods
Time:10:30 AM TO 01:30 PM   **Total Marks: 70**

Instructions:
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1 **(a)** State De Moivre's theorem. Find the roots of the equation $z^4 + 1 = 0$. **03**

(b) Find the real and imaginary parts of $f(z) = \sin z$ and check whether Cauchy Riemann equations are satisfied at all points. Hence find $f'(z)$. **04**

(c) (i) Use Lagrange's interpolation formula to estimate the value of y at $x = 6$ from the following data: **03**

x	3	7	9	11
y	168	120	72	48

(ii) Rearrange the following system of equations in diagonally dominant form and then solve the new system using Gauss Siedel Method: **04**
$$x + 20\,y + z = 44, \quad x + y + 20\,z = 63, \quad 20\,x + y + z = 25.$$

Q.2 **(a)** Perform four iterations of Bisection Method to find a root of **03**
$$f(x) = x^3 - 4x + 1 = 0.$$

(b) Find the bilinear transformation which maps $z = 0, 1, \infty$ into the points $w = i, 1, -i$. **04**

(c) Evaluate $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)}$ using contour integration. **07**

OR

(c) (i) Show that the function $f(x, y) = 2xy + i(x^2 - y^2)$ is differentiable at a single point $z = 0$. Hence find $f'(0)$. **03**

(ii) Prove that $u(x, y) = x^2 - y^2 + e^{-2x}\cos 2y$ is harmonic and find its harmonic conjugate. **04**

Q.3 **(a)** Evaluate $\oint_C \frac{z}{z^2+1}\,dz$ if C is the circle $|z + i| = 1$. **03**

(b) Find the real and imaginary part of $(1 + i)^i$. **04**

(c) Evaluate $\int_0^{1+i}(x^2 + iy)dz$ along the path $(i)\ y = x,\ (ii)\ y = x^2$. Is the line integral independent of the path? **07**

OR

Q.3 **(a)** Find the type of singularity of the function $f(z) = \frac{\sin z - z}{z^3}$. **03**

(b) Find and sketch the image of $|z + 1| = 1$ under the transformation $w = \frac{1}{z}$. **04**

(c) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in the Laurent series valid in the regions **07**
$$(i)\ |z| > 3 \qquad (ii)\ 1 < |z| < 3.$$

Q.4 **(a)** If $f(z)$ and $\overline{f(z)}$ are both analytic, then prove that $f(z)$ is constant. **03**

(b) Use modified Euler's method to find $y(0.2)$ given that **04**
$$\frac{dy}{dx} = 1 - y, \qquad y(0) = 0, \qquad h = 0.1.$$

(c) (i) Find a root of the equation $x = \cos x$ using Newton-Raphson Method correct up to four decimal places. Take $x_0 = 1$. **03**

(ii) Use Newton's divided difference formula to determine the polynomial **04**
which fits with the following data:

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

OR

Q.4 **(a)** If $f(z)$ is analytic, then prove that **03**
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|Re\, f(z)|^2 = 2\,|f'(z)|^2.$$

(b) Solve the following system of equations using Gauss Elimination Method: **04**
$$x + y + z = 3, \qquad x - y + z = 1, \qquad -x + y + 2z = 2.$$

(c) (i) Find the real root of the equation $2x - log_{10}^x = 7$ in $(3.5, 4.0)$ using False **03**
Position Method correct to three decimal places.

(ii) The population (in thousands) of a town is given below. Estimate the **04**
population for the year 1975 using interpolation.

Year	1971	1981	1991	2001	2011
Population	46	66	81	93	101

Q.5 **(a)** Evaluate $\int_0^1 xe^{-x}dx$ by using Gauss quadrature formula with $n = 3$. **03**

(b) Use fourth order Runge-Kutta Method to compute $y(0.1)$ and $y(0.2)$ given that **04**
$$\frac{dy}{dx} = y + \frac{4x}{y}, \quad y(0) = 1, \quad h = 0.1.$$

(c) (i) Evaluate $\int_4^{5.2} log\, x\, dx$ using Simpson's 3/8 rule. Take $h = 0.2$. **03**

(ii) Find the dominant Eigenvalue of $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ by Power Method and the **04**
corresponding Eigenvector.

OR

Q.5 **(a)** The following table gives the velocity v of a particle at time t: **03**

$t\ (sec.)$	0	2	4	6	8	10	12
$v\ (m\,/sec.)$	4	6	16	34	60	94	135

Find the distance moved by the particle in 12 seconds.

(b) Find an approximate value of y when $x = 0.1$ using modified Euler's method if **04**
$$\frac{dy}{dx} = x^2 + y, \quad y(0) = 1, \quad h = 0.05.$$
Perform two corrections at every step.

(c) Prove the following identities: **07**
$(i)\ \Delta + \nabla = \dfrac{\Delta}{\nabla} - \dfrac{\nabla}{\Delta}$

$(ii)\ 1 + \mu^2\delta^2 = \left(1 + \dfrac{1}{2}\delta^2\right)^2$

$(iii)\ \mu\delta = \dfrac{1}{2}\Delta E^{-1} + \dfrac{1}{2}\Delta.$

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