Seat No.: ________   Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 EXAMINATION – WINTER 2021

Subject Code:2110015   **Date:21/03/2022**

Subject Name:Vector Calculus And Linear Algebra

Time:10:30 AM TO 01:30 PM   **Total Marks:70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 (a) Objective Question (MCQ) **07**

1. Let A be a unitary matrix then A^{-1} is

   (A) A (B) $\overline{A}$ (C) A^T (D) $(\overline{A})^T$

2. For which value of k, u = (2,1,3) and v = (1,7,k) are orthogonal?

   (A) k =1 (B) k=3 (C) k =2 (D)None of these

3. The dimension of the solution space of $x - y = 0$ is

   (A) 1 (B) 2 (C) 3 (D) 4

4. The mapping $T: R^3 \rightarrow R^3$ defined by $T(v_1, v_2, v_3) = (v_1, 0, v_3)$ is called as

   (A) Reflection (B) Magnification (C) Rotation (D) Projection

5. Let $T: R^2 \rightarrow R^2$ be linear operator defined by $T(x, y) = (2x - y, -8x + 4y)$, what is basis for $\ker(T)$?

   (A) $\begin{bmatrix}2\\1\end{bmatrix}$ (B) $\begin{bmatrix}1\\1\end{bmatrix}$ (C) $\begin{bmatrix}1/2\\1\end{bmatrix}$ (D) None of these

6. What is angle between two vectors u= (1,0,1,0) and v = (-3,-3,-3,-3)?

   (A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

7. The column vectors of an orthogonal matrix are

   (A) Orthogonal (B)Orthonormal (C) Dependent (D)None of these

(b) Objective Question (MCQ) **07**

1. What is an eigenvalue of $A = \begin{bmatrix}3 & 2\\3 & 8\end{bmatrix}$?

   (A) 8 (B) 2 (C) 3 (D) 1

2. Cayley Hamilton theorem holds for _______ matrices only.

   (A) Singular (B) All square (C) Null (D)A few rectangular

3. If $r = xi + yj + zk$ then $\nabla r =$_____.

   (A) r (B)$\hat{r}$ (C) r^2 (D) None of these

4. $v = (x + 3y)i + (y - 2z)j + (x - 2z)k$ is _______?

   (A) Solenoidal (B) Irrotational (C) Both (D) None of these

5. The matrix of a quadratic form is a _________ matrix.

   (A) Symmetric (B)Skew symmetric (C) Hermitian (D) Skew hermitian

6. The value of the line integral $\int \nabla(x + y - z)$ from (0,1,-1) to (1,2,0) is____.

   (A)-1 (B) 3 (C) 0 (D) None of these

7. The value of line integral $\int_C \overline{F}\, dr$ does not depend on path C then $\overline{F}$ is

   (A) Solenoidal (B)Incompressible (C) Irrotational (D) None of these

Q.2	**(a)**	Find the rank of the matrix $A = \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$	**03**
	(b)	Find the inverse of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ using Row operation.	**04** **07**
	(c)	Determine whether the set R^+ of all positive real number with $x + y = xy$ and $kx = x^k$ is a vector space.	
Q.3	**(a)**	Does $W = \{(x, y, z) : x^2 + y^2 + z^2 \le 1\}$ is a subspace of R^3 with standard operation?	**03**
	(b)	Check whether the set $\{(2,1,1), (1,2,2), (1,1,1)\}$is linearly independent or linearly dependent in R^3.	**04**
	(c)	Determine the dimension and a basis for the solution space of the system of equation: $x + y - 2z = 0, -2x - 2y + 4z = 0, -x - y + 2z = 0$	**07**
Q.4	**(a)**	Show that the transformation $T: R^3 \to R^3$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + x_3, 3x_1 + x_2 - x_3, 2x_1 + 2x_2 + x_3)$ is linear.	**03**
	(b)	Consider the basis $S = \{v_1, v_2\}$, $v_1 = (-2,1), v_2 = (1,3)$ of R^2 and $T: R^2 \to R^3$ be the linear transformation such that $T(v_1) = (-1,2,0)$ and $T(v_2) = (0,-3,5)$. Find $T(x_1, x_2)$.	**04**
	(c)	Let R^3 have inner product $\langle (x_1, x_2, x_3) \cdot (y_1, y_2, y_3) \rangle = x_1y_1 + 2x_2y_2 + 3x_3y_3$. Use the Gram Schmidt process to transform the basis vectors $u_1(1,1,1), u_2(1,1,0)$ $u_3(1,0,0)$ into an orthonormal basis.	**07**
Q.5	**(a)**	Determine algebraic and geometric multiplicity of each eigen value of $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$	**03**
	(b)	Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ using Caley –Hamilton theorem.	**04**
	(c)	Find the matrix P such that diagonalizes the matrix $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and hence determine $P^{-1}AP$.	**07**
Q.6	**(a)**	Find the gradient of $f(x, y, z) = 2z^3 - 3(x^2 + y^2)z + tan^{-1}(xz)$ at (1,1,1).	**03**
	(b)	Prove that $F = (y^2 cosx + z^3)i + (2ysinx - 4)j + (3xz^2)k$, is irrotational and find its scalar potential.	**04**
	(c)	Find the least square solution of the linear system $Ax = b$, given by $x_1 + x_2 = 7, -x_1 + x_2 = 0, -x_1 + 2x_2 = -7$, and find the orthogonal projection of b on the column space of A.	**07**
Q.7	**(a)**	Find the work done by $\overline{F} = 5zi + y^2j + 10xk$ $over$ the $curve$ $r(t) = (sint)i + (cost)j + \left(\frac{t}{5}\right)k, where$ $0 \le t \le 2\pi$.	**03**
	(b)	Evaluate $\int_C \overline{F} dr$ along the parabola $y^2 = x$ between the points$(0,0)$ and $(1,1)$ $where$ $\overline{F} = x^2i + xyj$	**04**
	(c)	Verify Green's theorem for the function $F = (x + y)i + 2xyj$ and C is the path rectangle in the xy plane bounded by $x = 0, y = 0$ $x = a, y = b$.	**07**

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