Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 EXAMINATION – WINTER 2021

Subject Code:2110014 Date:19/03/2022

Subject Name:Calculus

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Marks

Q.1 (A) In each of the following choose the correct option. (MCQ) **07**

1. If the equation of the curve remains unchanged when x is replaces by – x, then the curve is symmetric about
   (a) origin (b) The line y = x (c) x – axis (d) y – axis

2. The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ is
   (a) convergent (b) divergent (c) oscillatory (d) None

3. The $\lim_{x\to 0} x^{\sin x} =$ ____
   (a) 0 (b) 1 (c) e (d) – 1

4. Let $a_1 = \sqrt{2}, a_{n+1} = \sqrt{2 + a_n}$ then the sequence $\{a_n\}$ converges to
   (a) 2 (b) 3 (c) $\sqrt{2}$ (d) 0

5. The radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ is
   (a) 1 (b) 2 (c) (0,1) (d) ∞

6. If $x = r\cos\theta, y = r\sin\theta$, then $\frac{\partial(x,y)}{\partial(r,\theta)}$ = _______
   (a) r (b) 1/r (c) $\sqrt{r}$ (d) None

7. Let $\int_0^1 f(x)\,dx$, then for which $f(x)$ given integral becomes improper ?
   (a) $(x + 1)^{-1}$ (b) $(x^2 + 1)^{-1}$ (c) x^{-1} (d) None

Q.1 (B) In each of the following choose the correct option. (MCQ) **07**

1. The limit of the series $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \cdots$ ..... is
   (a) log 2 (b) cos 1 (c) sin 1 (d) tan 1

2. If $u = f(xy/(x^2 + y^2))$, then $xu_x + yu_y =$ ___
   (a) 0 (b) u (c) 2u (d) – u

3. The equation of the tangent plane to the surface f(x,y,z) = xyz, at the point P(1,1,1) is
   (a) x + y – z =3 (b) x – y – z = 3 (c) x + y + z = 3 (d) x + y + z = 0

4. The $\lim_{x\to 0}(cosec\,x - x^{-1})$ is of the form _____

(a) 0/0 (b) ∞/∞ (c) 0^0 (d) $\infty - \infty$

5. The area of the region bounded by the parabolas $y = x^2$ and $x = y^2$ is

(a) 2/3 (b) –2/3 (c) 1/3 (d) –1/3

6. If $y = \int_1^{x^2} \cos t\, dt$ then $\frac{dy}{dt} =$ ___

(a) 2x cos x^2 (b) 2 (c) cos x (d) sin x

7. The series $\sum_{n=1}^{\infty} \frac{x^n}{n!}$ Converges for the values of x in the interval

(a) $(-\infty,\infty)$ (b) $(0,\infty)$ (c) $(-\infty,0)$ (d) $(-1,1)$

Q.2 (a) Show that $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges for p > 1. **03**

(b) Evaluate the following limits using L'Hospital's Rule: **04**

$(1)\ \lim_{x\to\pi/2}(\sec x - \tan x)$, $(2)\ \lim_{x\to\pi/2}(\tan x)^{\cos x}$.

(c) If $u = \sin^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then find the values of **07**

(i) $xu_x + yu_y$ and

(ii) $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$

Q.3 (a) Find the local extreme values of the function **03**

$f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4.$

(b) The lengths a, b, and c of the edges of a rectangular box are changing with time. At the instant a = 1 m, b = 2 m, c = 3 m, $\frac{da}{dt} = \frac{db}{dt} = 1$ m/sec, and $\frac{dc}{dt} = -3$ m/sec . At what rates are the box's volume V and surface area A changing at that instant? **04**

(c) A delivery company accepts only rectangular boxes the sum of whose length and girth (perimeter of a cross section) does not exceed 108 inch. Find the dimensions of an acceptable box of largest volume using Lagrange's multipliers method. **07**

Q.4 (a) Evaluate: **03**

$$\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz\, dy\, dx.$$

(b) Evaluate by reversing the order of the integration: **04**

$$\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y}\, dy\, dx$$

(c) Evaluate using the Polar Coordinates: **07**

(1) $\iint_R e^{x^2+y^2}\, dA$, where R is the semicircular region bounded by the x- axis and the curve $y = \sqrt{1-x^2}$.

(2) $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$.

Q.5 **(a)** Discuss the convergence of **03**

$$\sum_{n=1}^{\infty} \frac{\log n}{n^3}$$

(b) Test the convergence of : **04**

(1) $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$, (2) $\sum_{n=1}^{\infty} \frac{(n)^n}{\left(2^{n^2}\right)}$

(c) Examine for which values of x, the following series is convergent. **07**

$$\frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} - \frac{x^5}{5} + \cdots \dots \dots$$

Q.6 **(a)** Discuss the continuity of the function **03**

$$f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2} & ,(x,y) \neq (0,0) \\ 0\ , & ,(x,y) = (0,0). \end{cases}$$

(b) Check the convergence of **04**

$$\int_1^\infty \frac{\sin^3 x}{x^3}\, dx$$

(c) Find the Taylor's series expansion of $f(x+h) = \sqrt{x+h}$ up to fourth term. Apply it to find $\sqrt{25.15}$. **07**

Q.7 **(a)** Using the method of slicing find the volume of the cone with height 4 cm and radius of the base 4 cm. **03**

(b) The region enclosed by the parabola $y = x^2 + 1$ and the straight line $y = 2x + 1$ is revolved about the x- axis. Find the volume of solid of revolution. **04**

(c) Trace the curve $x^3 + y^3 = 3axy$, $a > 0$. **07**

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