Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–IV (NEW) EXAMINATION – WINTER 2020

Subject Code:2141905 Date:09/02/2021

Subject Name:Complex Variables and Numerical Methods

Time:02:30 PM TO 04:30 PM Total Marks:47

Instructions:

1. **Attempt any THREE questions from Q.1 to Q.6.**
2. **Q.7 is compulsory.**
3. **Make suitable assumptions wherever necessary.**
4. **Figures to the right indicate full marks.**

			MARKS
Q.1	**(a)**	Separate real and imaginary parts of $f(z) = e^{(z+2)}$, and also prove that it is analytic everywhere.	**03**
	(b)	Use De Moiver's theorem and find 4th root of unity in the complex plane.	**04**
	(c)	Use Gauss-Jacobi method to determine roots of the following equations $20x + y - 2z = 17$ $3x + 20y - z = -18$ $2x - 3y + 20z = 25$	**07**
Q.2	**(a)**	Evaluate the following integral along the curve $z(t) = t + it^2$ $\int_0^{2+4i} Re(z)dz$	**03**
	(b)	Evaluate $\oint \frac{\cos \pi z}{z-1} dz$ where C is the circle 1) $\lvert z\rvert = 2$ 2) $\lvert z\rvert = 1/2$	**04**
	(c)	Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane and finds it's conjugate harmonic function v.	**07**
Q.3	**(a)**	Obtain the Taylor's series of $f(z) = \sin z$ in powers of $\left(z - \frac{\pi}{4}\right)$.	**03**
	(b)	Find the center and radius of convergence of the power series $\sum_{n=0}^{\infty}(n + 2i)^n z^n$.	**04**
	(c)	Find the Laurent's series expansion of $f(z) = \frac{1}{(z+1)(z-2)}$ in the region 1) $1 < \lvert z\rvert < 2$ 2) $\lvert z\rvert > 2$	**07**
Q.4	**(a)**	Find the Maclaurin's series of $f(z)=sin^2 z$	**03**
	(b)	Find all values of z such that $e^z = 1 + i$	**04**
	(c)	Evaluate $\oint \frac{\cos z}{z^2-4} dz$ counterclockwise around C: $\lvert z\rvert = 5/2$	**07**
Q.5	**(a)**	Use Bisection method to find the real root of $x^3 - 4x - 9 = 0$. (Do 4 iterations)	**03**
	(b)	Using Newton's divided difference interpolation formula, compute $f(10.5)$ from the following data:	**04**

x	10	11	13	17
$f(x)$	2.3026	2.3979	2.5649	2.8332

(c) Use Simpson's 3/8 rule and evaluate the following integral taking n=6, and hence calculate $\log_e 2$. Also, find the error involved in the pross. **07**

$$\int_0^3 \frac{dx}{1+x}$$

Q.6 **(a)** Approximate the root of the equation $e^x - 2\cos x = 0$, by three iterations of Newton Raphson method, taking initial approximation as $x_0 = 2$. **03**

(b) Find an approximate value of $f(3.6)$ using Newton's backward difference formula from the following data: **04**

x	0	1	2	3	4
$f(x)$	-5	1	9	25	55

(c) Using power method, determine the largest eigenvalue and the corresponding eigenvector of the matrix $= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, taking initial eigenvector $x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. **07**

Q.7 Using three point Gaussian formula evaluate the following integral and compare with the exact value. **05**

$$\int_{-1}^{1} \frac{dx}{1+x^2}$$

OR

Q.7 Solve the following system of linear equations using Gauss Elimination Method. **05**

$x + y + z = 9; 2x - 3y + 4z = 13; 3x + 4y + 5z = 40$

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