Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II (NEW) EXAMINATION – WINTER 2020

Subject Code:2110014 **Date:16/03/2021**

Subject Name:Calculus

Time:10:30 AM TO 12:30 PM **Total Marks:47**

Instructions:

1. **Attempt any SIX questions from Q1 to Q12.**
2. **Q13 is compulsory.**
3. **Make suitable assumptions wherever necessary.**
4. **Figures to the right indicate full marks.**

MARKS

Q.1 Objective Questions (MCQS)

(a) **07**

1. The value of $\lim_{n\to\infty} (n)^{\frac{1}{n}}$ is

(a) 1 (b) 0 (c) e (d) ∞

2. $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + ...$ represents Maclaurine series expansion of function

(a) $sin\,x$ (b) $cos\,x$ (c) $cos\,hx$ (d) $sin\,hx$

3. The value of $\lim_{x\to\infty} \frac{log\ x}{x^n}$ $for\ n > 0\ is$

(a) 1 (b) 0 (c) -1 (d) ∞

4. The value of $\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx$ is

(a) 1 (b) 0 (c) e (d) ∞

5. The value of $\lim_{(x,y)\to(1,2)} \frac{x^2 + y}{3x + y^2}$ is

(a) 1/2 (b) 0 (c) 3/7 (d) 7/3

6. $f(x, y) = x^3\ \tan\left(\frac{y}{x}\right)$ is a homogeneous function of degree

(a) 0 (b) 3 (c) -3 (d) 2

7. The curve $y^2(2a - x) = x^3$ is symmetric about

(a) $x-axis$ (b) $y-axis$ (c) origin (d) $x = y\ line$

Q.2 Objective Questions (MCQS) **07**

1. The value of $\lim_{x\to 0} x\,(log\ x)$ is

(a) 1 (b) 0 (c) -1 (d) ∞

2. Find $\lim_{(x,y)\to(0,0)} \frac{x - y}{x + y}$

(a) 1 (b) -1 (c) e (d) Limit does not exist

3. If $u = sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = ?$

(a) $sin\,u$ (b) $tan\,u$ (c) $-sin\,u$ (d) $-tan\,u$

4. The curve $a^2x^2 = y^3(2a - y)$ is symmetric about

(a) $x-axis$ (b) $y-axis$ (c) origin (d) $x = y\ line$

5. If $x = r\cos\theta$ and $y = r\sin\theta$ then $\dfrac{\partial(x,y)}{\partial(r,\theta)} = ?$

(a) $r\cos\theta$ (b) $r\sin\theta$ (c) r (d) $-r$

6. $1+2+3+4+5+.....+n = ?$

(a) $\dfrac{n(n+1)}{2}$ (b) $\dfrac{n(n+1)(2n+1)}{6}$ (c) $\dfrac{n(n-1)}{2}$ (d) $n(n+1)$

7. Test the convergence of $\sum_{n=0}^{\infty} \dfrac{2^n}{3^n}$

(a) Convergent (b) Divergent (c) Finitely Oscillating (d) Infinitely Oscillating

Q.3 **(a)** Find the equations of the tangent plane and normal line to the surface $x^2+2y^2+3z^2=12$ at $(1, 2, -1)$. **03**

(b) If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, Prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. **04**

Q.4 If $u = \sin^{-1}\left[\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right]$ then using Euler's Theorem prove the following statements **07**

(1) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{2}\tan u$

(2) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\,\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \dfrac{1}{4}\dfrac{\sin u\,(1-2\cos^2 u)}{\cos^3 u}$.

Q.5 **(a)** Expand $x^2 y + 3y - 2$ in terms of $(x-1)$ and $(y+2)$ **03**

(b) Find the Maximum and Minimum values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ **04**

Q.6 Trace the curve (a) $xy^2 = a^2(a-x)$ (b) $r = a(1+\cos\theta)$ **07**

Q.7 **(a)** Show that the series $1 - \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} - \dfrac{1}{\sqrt{4}} + ...\infty$ is Convergent. **03**

(b) Test the convergence of the series $\sum_{n=1}^{\infty} \dfrac{2^n+1}{3^n+1}$ **04**

Q.8 Test the convergence of the series $\sum \dfrac{(n+1)^n x^n}{n^{n+1}}$ **07**

Q.9 **(a)** Evaluate $\lim_{x\to 0} \dfrac{\log(\sin x)}{(\cot x)}$. **03**

(b) Define the third type of Improper Integral and Evaluate $\int_0^{\infty} \dfrac{1}{x^2}\,dx$ **04**

Q.10 Find the Volume of the solid obtained by rotating the region enclosed by the curves $y = x$ & $y = x^2$ about the x axis. **07**

Q.11 **(a)** Change the order of the integration of $\int_0^1 \int_x^{\sqrt{x}} f(x,y)\, dy\, dx$ **03**

(b) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} dz\, dy\, dx$ **04**

Q.12 Evaluate $\iiint_E 2x\, dV$ where E is the region under the plane $2x+3y+z=6$ that lies in the first octane. **07**

Q.13 Prove that $1+\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\frac{16}{81}+..........$ converges and Find its sum. **05**

OR

Q.13 Find the volume of the tetrahedron bounded by the plane $x+y+z=2$ and the planes $x=0,\ y=0,\ z=0$ **05**

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