Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– VI (New) EXAMINATION – WINTER 2019

Subject Code: 2160704 Date: 09/12/2019

Subject Name: Theory of Computation

Time: 02:30 PM TO 05:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

MARKS

Q.1 **(a)** Define – bijection function. Check whether the function $f : Z \rightarrow Z$ defined by $f(x) = 2x$ is a bijection function or not. Justify your answer. **03**

(b) Draw an FA that recognizes the language of all strings containing even no of 0's and even no of 1's over $\sum = \{0,1\}$. Also write a regular expression for the same language. **04**

(c) Write the principle of Mathematical Induction. Prove using mathematical induction that for every $n \geq 0$, **07**

$$\sum_{i=1}^{n} \frac{1}{i\,(i+1)} = \frac{n}{n+1}$$

(Consider the sum on the left is 0 for n = 0)

Q.2 **(a)** Find regular expression and also derive the words corresponding to the language defined recursively below over $\sum = \{a, b\}$. **03**

i. $a \in L$

ii. For any $x \in L$, xa and xb are elements of L

(b) Define – Equivalence relation. A relation on the set {1,2,3} is given as $R = \{(a, b) \mid a - b \text{ is an even no}\}$. Check whether R is equivalence relation or not. Give reasons. **04**

(c) Give transition table for PDA recognizing the following language and trace the move of the machine for input string abcba: **07**

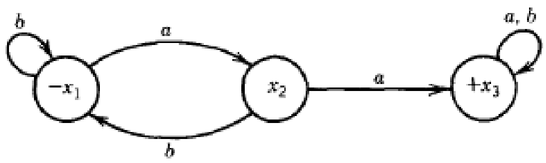
$L = \{xcx^r \mid x \in \{a, b\}^*\}$

OR

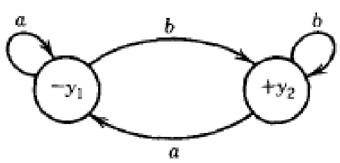
(c) Give transition table for PDA accepting the language of all odd-length strings over {a, b} with middle symbol a. Also draw a PDA for the same. **07**

Q.3 **(a)** Let FA_1 and FA_2 be the FAs as shown in the figure recognizing the languages L_1 and L_2 respectively. Draw an FA recognizing the language, $L_1 \cup L_2$. **03**

FA_1 :



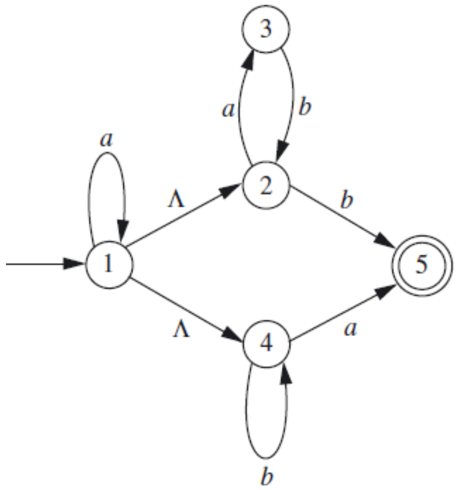
FA_2 :



(b) Define – Moore machine. Convert the following Moore machine into its equivalent Mealy machine: **04**

Old state	After input a	After input b	Output
	New state	New state	
$-q_0$	q_1	q_2	0
q_1	q_3	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_3	1

(c) Convert the following NFA - Λ into its equivalent DFA that accepts the same language: **07**



OR

Q.3 **(a)** Prove that – "If there is a CFG for the language L that has no Λ-productions, then there is a CFG for L with no Λ-productions and no unit productions". Support your answer with the help of the following CFG: **03**

S → A | bb
A → B | b
B → S | a

(b) Write CFG for the following languages : **04**

i. $\{a^i b^j c^k \mid i = j + k\}$

ii. $\{a^i b^j c^k \mid j = i \text{ or } j = k\}$

(c) Define – ambiguous grammar, leftmost derivation. Check whether the following grammars are ambiguous or not. Justify your answer with proper reason. **07**

i.
S → ABA
A → aA | Λ
B → bB | Λ

ii.
S → A | B
A → aAb | aabb
B → abB | Λ

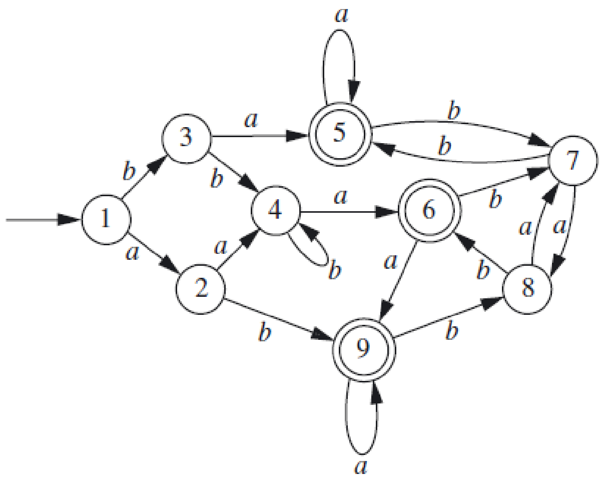
Q.4 **(a)** Describe the language generated by the following grammars: **03**

i.
S → aA | bC | b
A → aS | bB
B → aC | bA | a
C → aB | bS

ii.
S → aT | bT | Λ
T → aS | bS

(b) Discuss – Nondeterministic Turing Machines and Universal Turing Machines **04**

(c) Find a minimum-state FA for the following FA that recognizes the same language using the minimization algorithm: **07**



OR

Q.4 **(a)** Find the CFG for the regular expression : $(011 + 1)^* \, (01)^*$ **03**

(b) Prove that the language $L = \{a^n b^n a b^{n+1} \mid n = 1, 2, 3, \ldots\}$ is nonregular using pumping lemma. **04**

(c) Convert the following CFG into its equivalent CNF: **07**

$S \to TU \mid V$
$T \to aTb \mid \Lambda$
$U \to cU \mid \Lambda$
$V \to aVc \mid W$
$W \to bW \mid \Lambda$

Q.5 **(a)** Convert the following CFG into its equivalent PDA. **03**

$S \to AB$
$A \to BB$
$B \to AB$
$A \to a$
$B \to a \mid b$

(b) Show using the pumping lemma that the following language is not a CFL. **04**

$L = \{a^i b^j c^k \mid i < j < k\}$

(c) Draw a Turing Machine that accepts the language $\{a^n b^n a^n \mid n \geq 0\}$ over $\{a, b\}^*$. Also trace the TM on input string aaabbbaaa. **07**

OR

Q.5 **(a)** Define Context Sensitive Language and Context Sensitive Grammar. Write CSG for $L = \{a^n \, b^n \, c^n \mid n \geq 1\}$. **03**

(b) Define - Primitive recursive functions and also give complete primitive recursive derivations for the function, $f : N \to N$ defined by $Add(x, y) = x + y$. **04**

(c) Draw a Turing Machine that accepts the language $\{xx \mid x \in \{a, b\}^*\}$. Also trace the TM on input string aa. **07**

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