Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– I & II (NEW) EXAMINATION – WINTER 2019

Subject Code: 2110015 **Date: 01/01/2020**

Subject Name: Vector Calculus And Linear Algebra

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q-1 (a) Objective Questions **Marks 07**

1. If $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 9 & 7 \\ 11 & 9 & 8 \end{bmatrix}$ then trace of the matrix A is

(a) 12 (b) 18 (c) 72 (d) 16

2. If $div\ u = 0$ then u is said to be

(a) Rotational (b) Solenoidal (c) Compressible (d) None of these

3. If A is 3×3 invertible matrix then nullity of A is

(a) 1 (b) 2 (c) 0 (d) 3

4. If matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ is having Eigen values 2,3,6 then Eigen values of A^{-1} are

(a) 2,3,6 (b) $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ (c) $1, \frac{2}{3}, \frac{1}{3}$ (d) None of these

5. Which set from $S_1 = \{(x, y, z) \in R^3 / z > 0\}$ and $S_2 = \{(x, y, z) \in R^3 / x = z = 0\}$ is subspace of R^3.

(a) S_1 (b) S_2 (c) S_1 and S_2 (d) None.

6 If Eigen values of 3×3 matrix A are 5,5,5 then Algebraic multiplicity of matrix A is

(a) 3 (b) 1 (c) 5 (d) 0.

7 For what values of c, the vector $(2, -1, c)$ has norm 3?

(a) -3 (b) 3 (c) 0 (d) 2

(b) Objective Questions **07**

1. If $F(x, y, z) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then $Curl\ \vec{F}$ is

(a) 1 (b) 3 (c) 2 (d) 0

2. For what value of k the vectors $v_1 = (-1,2,4)$, $v_2 = (-3,6,k)$ are Linearly Dependent?

(a) 12 (b) 7 (c) 4 (d) 1

3. Which one is the characteristic equation of $= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$?

(a) $\lambda^2 - 5\lambda + 4 = 0$ (b) $\lambda^2 - 4\lambda - 5 = 0$

(c) $\lambda^2 + 4\lambda + 5 = 0$ (d) $\lambda^2 + 5\lambda + 4 = 0$

4. Which matrix represents one to one transformation

(a) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 7 \\ 4 & 14 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 7 \\ 1 & 14 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

5. Matrix $A = \begin{bmatrix} i & 2+3i \\ 2-3i & 0 \end{bmatrix}$ is

(a) Symmetric (b) Skew-symmetric (c) Hermitian (d) None of these

6. If u and v are vectors in an Inner product space then

(a) $|\langle u, v\rangle| = \|u\|\|v\|$ (b) $|\langle u, v\rangle| \leq \|u\|\|v\|$

(c) $|\langle u, v\rangle| \geq \|u\|\|v\|$ (d) None of these.

7. Each vector in R^2can be rotated in counter clockwise direction with 90° is followed by the matrix,

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

Q-2 **(a)** Find the Rank of a Matrix $A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 2 & 1 & -2 & -2 \\ -1 & 2 & -4 & 1 \\ 3 & 0 & 0 & -3 \end{bmatrix}$ **03**

(b) Determine whether the given vectors $v_1 = (2,-1,3)$; $v_2 = (4,1,2)$; $v_3 = (8,-1,8)$ Span R^3. **04**

(c) For which values of 'a' will the following system have no solutions? Exactly one solution? Infinitely many solutions? **07**

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2 - 14)z = a + 2$$

Q-3 **(a)** Define Singular Matrix. Find the inverse of the matrix A using Gauss Jordan Method if it is invertible $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ **03**

(b) Express the matrix $A = \begin{bmatrix} 1 & 5 & 7 \\ -1 & -2 & -4 \\ 8 & 2 & 13 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix **04**

(c) Fine a basis for the nullspace, row space and column space of the matrix **07**

$$A = \begin{bmatrix} -1 & 2 & -1 & 5 & 6 \\ 4 & -4 & -4 & -12 & -8 \\ 2 & 0 & -6 & -2 & 4 \\ -3 & 1 & 7 & -2 & 12 \end{bmatrix}$$

Also determine rank and nullity of the matrix.

Q-4 **(a)** Let $T_1: R^2 \to R^2$, $T_2: R^2 \to R^3$ be transformation given by **03**

$T_1(x, y) = (x + y, \ y)$ and $T_2(x, y) = (2x, \ y, \ x + y)$.

Show that T_1 is linear transformation and also find formula for $T_2 \circ T_1$.

(b) Let $T: R^2 \to R^2$ be the linear operator defined by **04**

$T(x,y) = (2x - y, -8x + 4y)$. Find a basis for $\ker(T)$ and basis for $R(T)$.

(c) Find a matrix P that diagonalize A, where $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ **07**

And determine $P^{-1}AP$.

Q-5 **(a)** Find constants a, b, c so that **03**

$v = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational.

(b) Let the vector space P_2 have the inner product $\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)\, dx$ **04**

(i) Find $\|p\|$ for $p = x^2$.

(ii) Find $d(p,q)$ of $p = 1$ and $q = x$.

(c) Using Gram-Schmidt process orthonormalize the set of linearly independent vectors $u_1 = (1,\ 0,\ 1,\ 1)$, $u_2 = (-1,\ 0, -1,\ 1)$ and $u_3 = (0, -1,\ 1,\ 1)$ of R^4 with standard inner product. **07**

Q-6 **(a)** The temperature at any point in space is given by $T = xy + yz + zx$. Determine the derivative of T in the direction of the vector $3\hat{\imath} - 4\hat{k}$ at the point $(1,1,1)$. **03**

(b) Find the orthogonal projection of $u = (2,1,3)$ on the subspace of R^3 spanned by the vectors $v_1 = (1,1,0)$, $v_2 = (1,2,1)$. **04**

(c) Verify Green's theorem for the field $F = (x - y)\hat{\imath} + x\hat{\jmath}$ and the region R bounded by the unit circle C: $r(t) = (\cos t)\hat{\imath} + (sint)\hat{\jmath}$; $0 \le t \le 2\pi$ **07**

Q-7 **(a)** Find the co ordinate vector of $p = 2 - x + x^2$ relative to the basis **03**

$S = \{p_1, p_2, p_3\}$ where $p_1 = 1 + x$, $p_2 = 1 + x^2$, $p_3 = x + x^2$

(b) Let $T: R^3 \to R^3$ be multiplication by A determine whether T has inverse. If so find $T^{-1}(x_1, x_2, x_3)$, where $A = \begin{bmatrix} 1 & 4 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ **04**

(c) Determine whether R^+ of all positive real numbers with operators **07**

$x + y = xy$ and $kx = x^k$ as a Vector Space.

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