

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-IV (NEW) EXAMINATION – WINTER 2018****Subject Code:2141905****Date:22/11/2018****Subject Name:Complex Variables and Numerical Methods****Time: 02:30 PM TO 05:30 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Find the roots of the equation $Z^2 + 2iz + (2 - 4i) = 0$ **03**
- (b) Show that $f(z) = zI_m(z)$ is differentiable only at $z = 0$ and $f'(0) = 0$. **04**
- (c) Solve the following system of equation by Gauss-Seidal method correct to three decimal places. **07**

$$\begin{aligned} 2x + y + 54z &= 110 \\ 27x + 6y - z &= 85 \\ 6x + 15y + 2z &= 72 \end{aligned}$$

- Q.2** (a) Evaluate $\int_0^{2+i} z^2 dz$ along the line joining the points $(0,0)$ and $(2,1)$. **03**
- (b) Determine the mobius transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively. **04**
- (c) Prove that the n^{th} roots of unity are in geometric progression. Also show that their sum is zero. **07**

OR

- (c) Verify that C-R equation are satisfied at $z = 0$ for the function $f(z) = \begin{cases} \frac{z^{-2}}{z} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ **07**
- Q.3** (a) Evaluate $\oint_C \left[\frac{3}{z-i} - \frac{6}{(z-i)^2} \right] dz$, where $C: |z| = 2$. **03**
- (b) Find the radius of convergence of $\sum_{n=1}^{\infty} \left(\frac{6n+1}{2n+5} \right)^2 (z - 2i)^n$ **04**
- (c) Using the residue theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\sin\theta}$ **07**

OR

- Q.3** (a) Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor's series about the point $z = 0$. **03**
- (b) Check whether $f(z) = \sin z$ is analytic or not. If analytic find its derivative. **04**
- (c) Evaluate $\oint_C \frac{z^3 - z^2 + z - 1}{z^3 + 4z} dz$ counter clockwise around C, where C is $|z| = 1$ and $|z| = 3$. **07**

- Q.4 (a)** Using Newton's forward formula, find the value of $f(1.6)$ if **03**

x	1	1.4	1.8	2.2
f(x)	3.49	4.82	5.96	6.5

- (b)** Find the Lagrange interpolating polynomial from the following data **04**

x	0	1	4	5
f(x)	1	3	24	39

- (c)** Find a root of $x^4 - x^3 + 10x + 7 = 0$ correct to three decimal places between $a = -2$ and $b = -1$ by Newton-Raphson method. **07**

OR

- Q.4 (a)** Solve the system of equation by Gauss elimination method. **03**

$$x + y + z = 9$$

$$2x - 3y + 4z = 13$$

$$3x + 4y + 5z = 40$$

- (b)** Compute $f(8)$ from the following values using Newton's Divided difference formula **04**

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

- (c)** Evaluate $\int_0^6 \frac{1}{1+x} dx$, taking $h = 1$ and using Simpson's $\frac{1}{3}$ rule. Hence obtain approximate value of $\log_8 7$. **07**

- Q.5 (a)** Evaluate $\Delta^n e^x$ **03**

- (b)** Use power method to find the largest of Eigen values of the matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ **04**

- (c)** Use Euler's method to obtain an approximate value of $y(0.4)$ for the differential equation $y' = x + y, y(0) = 1$ with $h = 0.1$. **07**

OR

- Q.5 (a)** Prove that $hD = \log(1 + \Delta)$ **03**

- (b)** Evaluate $I = \int_{-1}^1 \frac{dx}{1+x^2}$ by one point, two point and three point Gaussian formula. **04**

- (c)** Determine $y(0.1), y(0.2)$ correct upto four decimal places by fourth order Runge-Kutta method from $\frac{dy}{dx} = 2x + y, y(0) = 1$ **07**
