

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER- 1st / 2nd EXAMINATION (NEW SYLLABUS) – WINTER 2018

Subject Code: 2110014

Date: 07/01/2019

Subject Name: Calculus

Time: 10:30 AM TO 1:30 PM

Total Marks: 70

Instructions:

1. Question No. 1 is compulsory. Attempt any four out of remaining Six questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Objective Question (MCQ)

(a)

07

1. $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} = \underline{\hspace{2cm}}$

- (a) 0 (b) 1 (c) 0.5 (d) doesn't exist

2. $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} dx dy dz = \underline{\hspace{2cm}}$

- (a) 1 (b) e^3 (c) $(e - 1)^3$ (d) $(1 - e)^3$

3. The jacobian of polar coordinates (r, θ) with respect to Cartesian coordinates (x, y) is

- (a) r (b) $r \sin \theta$ (c) r^2 (d) None of these

4. The curve $x^2 + y = 1$ is symmetric about _____

- (a) X - axis (b) Y - axis (c) Origin (d) None of these

5. $\lim_{n \rightarrow \infty} \left(\frac{n+4}{n} \right)^n$

- (a) 1 (b) e (c) e^3 (d) e^4

6. Which of following series diverges?

(a) $\sum_{n=0}^{\infty} \frac{5}{2^n}$ (b) $\sum_{n=1}^{\infty} \frac{1}{9} \left(\frac{1}{3} \right)^{n-1}$ (c) $\sum_{n=1}^{\infty} \frac{1}{n}$ (d) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

7. The improper Integral $\int_1^{\infty} \frac{1}{x} dx$ is

- (a) 0 (b) 1 (c) 2 (d) Diverges

(b)

07

1. If $u(x, y) = \tan^{-1}(\log x - \log y)$ then $x u_x + y u_y =$

- (a) u (b) 0 (c) $-u$ (d) $2u$

2. $\lim_{(x,y) \rightarrow (0,0)} \frac{x-y}{x+y}$ is

- (a) 0 (b) -1 (c) 1 (d) doesn't exist

3. Which of the following is homogeneous function of degree one?

- (a) $\ln x - \ln y$ (b) $x^2 y + x^3$ (c) $\tan(x^2 + y^2)$ (d) $(x^2 + y^2)/(x + y)$

4. The Maclaurin's Series expansion of the function e^{6x} has coefficient of x^3

- (a) 6 (b) $3!$ (c) 36 (d) $1/3!$

5. The radius of convergence of the series $1+2+4+8+\dots+2^n+\dots$

- (a) 0 (b) 1 (c) 2 (d) ∞

6. Standard linear approximation of $f(x,y) = xyz$ at $(1,1,1)$ is
 (a) $x+y+z-2$ (b) $x+y+z+2$ (c) $x-y+z+2$ (d) None of these
7. The stationary point of the function $f(x,y) = x^2 + xy + y^2$ is
 (a) $(1,0)$ (b) $(1,1)$ (c) $(0,0)$ (d) $(0,-1)$
- Q.2** (a) Find the value of b for which $1 + e^b + e^{2b} + \dots + e^{nb} + \dots = 9$. **03**
- (b) Evaluate (i) $\lim_{n \rightarrow \infty} \frac{\ln n}{n}$ (ii) $\lim_{x \rightarrow 0} \left(\frac{1}{x}\right)^{1-\cos x}$ **04**
- (c) Find Taylor series expansion of the function $\sqrt{x+h}$ in powers of h and hence find the value of $\sqrt{18}$ correct up to three decimal places. **07**
- Q.3** (a) Determine whether $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^4 + 3y^4}$ exists or not? If they exist, find the value. **03**
- (b) If $z(x+y) = x^2 + y^2$, show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$. **04**
- (c) State Euler's theorem for homogeneous function. **07**
 Verify Euler's theorem for $f(x,y) = (x^3 + y^3)/(x+y)$.
- Q.4** (a) The radius of a sphere is found to be 10 cm with a possible error of 0.02 cm. What is the relative error in computing the volume? **03**
- (b) Find tangent plane and normal line of the surface $x^2 + y^2 + z = 9$ at the point $P(1,2,4)$. **04**
- (c) Find the numbers x, y and z such that $xyz = 16$ and $18xy + 16yz + 12xz$ is minimum, using the Lagrange's method of undetermined coefficients. **07**
- Q.5** (a) Find the interval of convergence of the series $1 + 2x + 3x^2 + 4x^3 + \dots$. **03**
- (b) Find the volume of the region D enclosed by surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$. **04**
- (c) Define jacobian. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y} (y-2x)^2 dy dx$ using change of variables $x+y = u$ and $y-2x = v$. **07**
- Q.6** (a) Show that the p -series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ (p a real constant) converges if $p > 1$. **03**
- (b) Evaluate $\int_0^{\infty} \int_{-y}^y (y^2 - x^2) e^{-y} dx dy$ by changing the order of integration. **04**
- (c) Test the convergence of the series. If converges find the sum. **07**
 (i) $\sum_{n=1}^{\infty} \tan^{-1}(n) - \tan^{-1}(n+1)$ (ii) $\sum_{n=0}^{\infty} \frac{1+2^n}{5^n}$,
- Q.7** (a) The region bounded between the graph of $y = x^2$ and $y = -x + 2$ and is rotating around the line $y = 4$. Find its volume. **03**
- (b) State the types of the improper integrals. Test the convergence of $\int_1^{\infty} \frac{1}{\sqrt{x^2-0.2}} dx$. **04**
- (c) Trace the curve $r = 1 - \cos \theta$. **07**