Enrolment No. /Seat No.: _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BCA/ MCA INTEGRATED – SEMESTER I- EXAMINATION –SUMMER-2025

Subject Code: BC01001051 **Date: 12/06/2025**

Subject Name: Mathematics-1

Time:02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make Suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Use of simple calculators and non-programmable scientific calculators are permitted.**

Q.1 **(a)** Define the following terms. **(7)**

(1) Unit matrix

(2) Power set

(3) Symmetric matrix

(4) Many - One Function

(5) Coordinate Geometry

(6) Explicit Function

(7) Quadrants

(b) (i) If $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $X = \{1, 2, 3\}$, $Y = \{2, 4\}$ $Z = \{1, 5, 9\}$ then find (i) $X \triangle Y$ (ii) $(X \cup Y) \cap Z$ (iii) $X' \cup Y'$ **(3)**

(ii) If $A = \begin{bmatrix} -1 & 4 \\ 3 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ 2 & 9 \end{bmatrix}$ then show that **(4)**

$(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$

Q.2 **(a)** (i) Verify $(A \cup B)' = A' \cap B'$ by Venn diagram **(3)**

(ii) If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{1, 3, 4\}$ and $D = \{2, 4, 5\}$, then verify that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$ **(4)**

(b) In a class of 42 students, each play at least one of the three games Cricket, Hockey and Football. It is found that 14 play Cricket, 20 play Hockey and 24 play Football, 3 play both Cricket and Football, 2 play both Hockey and Football. None play all the three games. Find the number of students who play Cricket but not Hockey. **(7)**

OR

(b) Define Difference of two sets. If $U = \{x/x \in N; x \leq 10\}$, $A = \{x/x \in N; x^2 < 10\}$, $B = \{2, 4, 6\}$, $C = \{x/x^3 - 3x^2 - 4x = 0\}$ Verify that (i) $A \cap (B - C) = (A \cap B) - (A \cap C)$ (ii) $A' - B' = A - B$ **(7)**

Q.3 **(a)** (i) $f: N \to N$ and $f(x) = 5x - 2$. If the range of function f is $\{3, 8, 13\}$ Find the domain of f. **(3)**

(ii) Examine whether the following functions are equal. **(4)**
$f(x) = \frac{x^2-9x+14}{x-2}$, $x \in Z - \{2\}$, and $g(x) = x - 7$, where $x \in Z - \{2\}$.

(b) The expenditure of a hostel depends upon the number of students. When there are 40 students the expenditure is Rs. 50,000 and for 50 students the expenditure is Rs. 62,000. If the relationship between the number of students and hostel expenditure is linear, find the relationship and also find the expenditure when there are 60 students in the hostel. **(7)**

OR

(a) (i) If $f(x) = x^2(x-1)^2$, $x \in R$, prove that $f(x+1) - f(x) = 4x^3$ **(3)**

(ii) Define Bijective function. If $f: R \to R$, $f(x) = x + 1$, show that f is bijective. **(4)**

(b) Let $f(x) = x + 2$, $g(x) = x - 2$, $h(x) = 3x$, for $x \in R$, where R is the set of real numbers. Find (i) $g \circ f$ (ii) $f \circ g$ (iii) $f \circ f$ (iv) $h \circ g$ (v) $f \circ h$ (vi) $h \circ f$ (vii) $f \circ h \circ g$ **(7)**

Q.4 **(a)** (i) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ then find $A^2 - 5A + 3I$. **(3)**

(ii) If $A = \begin{bmatrix} 5 & -4 & 0 \\ 2 & -2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 2 & 3 \\ 8 & 5 & 0 \\ 1 & 1 & -2 \end{bmatrix}$ then verify that **(4)**
$(A + B)^T = A^T + B^T$

(b) Find the inverse of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ **(7)**

OR

(a) (i) If $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$, Find $A + A^T + A^{-1}$ **(3)**

(ii) Solve the following equations, using Cramer's rule **(4)**
$2x + 3y = 13$
$x + y = 5$

(b) Define Rank of a matrix. **(7)**
Find the rank of the matrix $A = \begin{bmatrix} 7 & -1 & 0 \\ 1 & 1 & 4 \\ 13 & -3 & -4 \end{bmatrix}$, Using row operation.

Q.5 **(a)** (i) Find the area of triangle whose vertices are $(2, 3)$, $(5, 7)$, $(-3, 4)$. **(3)**

(ii) Find the coordinates of the point which divides the points $P(8, 9)$ and $Q(-7, 4)$ internally in the ratio $2:3$ and externally in the ratio $4:3$. **(4)**

(b) Prove that the quadrilateral with vertices $(2, -1)$, $(3, 4)$, $(-2, 3)$ and $(-3, -2)$ is a rhombus. **(7)**

OR

(a) (i) Find the equation of the line having the points $(1, 2)$ and $(-2, 0)$. **(3)**

(ii) For what values of k, the lines $3x + 2y + 7 = 0$ and $2x + ky + 9 = 0$ will be perpendicular to each other ? **(4)**

(b) Find the angle between the lines $2x + y - 3 = 0$ and $x + 3y + 2 = 0$. **(7)**

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