Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV EXAMINATION – SUMMER 2025

Subject Code:2141905 Date:08-05-2025

Subject Name:Complex Variables and Numerical Methods

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	Discuss continuity of $f(z) = \begin{cases} \frac{Re(z^2)}{\lvert z\rvert^2} & ; z \neq 0 \\ 0 & ; z = 0 \end{cases}$ at z = 0.	**3**
	(b)	Find the analytic function $f(z) = u + iv$, if $u - v = e^x(cosy - siny)$.	**4**
	(c)	Find the Laurent's series expansion of $f(z) = \frac{1}{z(z^2-3z+2)}$ about z = 0, for the regions (i) $1 < \lvert z\rvert < 2$ (ii) $\lvert z\rvert > 2$.	**7**
Q.2	**(a)**	Evaluate the integral $\int_{-1}^{1} \frac{dx}{1+x^2}$ by Gaussian integration two pint formula.	**3**
	(b)	Evaluate $\int_{0.2}^{1.4}(sinx - logx + e^x)dx$ with h=0.2 by simpson's 1/3rd and 3/8th rule.	**4**
	(c)	Use the power method to find the largest eigen value and corresponding eigen vector of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$	**7**
		OR	
	(c)	Solve the following system of equations using Gauss seidel method: $5x + y - z = 10$; $2x + 4y + z = 14$; $x + y + 8z = 20$	**7**
Q.3	**(a)**	Use secant method to estimate the root of $cosx = xe^x$ correct to four significant digits	**3**
	(b)	Find a real root of $xe^x = 2$, correct up three decimal places, by using Newton-raphson method	**4**
	(c)	Compute f (9.2) from the following value Newton's divided difference formula.	**7**

X	8.0	9.0	9.5	11
F(x)	2.079442	2.197225	2.251292	2.397895

OR

Q.3	**(a)**	Use trapezoidal rule to evaluate $\int_0^1 x^3 dx$ considering five sub intervals.	**3**
	(b)	Using Newton's forward formula, find the value of f(1.6) if	**4**

X	1	1.4	1.8	2.2
F(x)	3.49	4.82	5.96	6.5

	(c)	Solve the following system of equation by gauss elimination method with partial pivoting $2x_1 + 2x_2 + x_3 = 6;\ 4x_1 + 2x_2 + 3x_3 = 4\ ;\ x_1 + x_2 + x_3 = 0$	**7**
Q.4	**(a)**	If $f(z) = u + iv$ is analytic in domain D then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\|Ref(z)|^2 = 2|f'(z)|^2$.	**3**
	(b)	Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and residue at each pole. Evaluate $\int_c f(z)dz,$ where c is the circle $\|z\| = 3$.	**4**
	(c)	Show that the function $f(z) = \sqrt{\|xy\|}$ satisfies the Cauchy- Riemann equations at the origin but $f'(0)$ fails to exits.	**7**
		OR	
Q.4	**(a)**	Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor's series about the point z = 1.	**3**
	(b)	Find the radius of converges of $\sum_{n=1}^{\infty}\left(\frac{6n+1}{2n+5}\right)^2 (z-2i)^n$.	**4**
	(c)	Simplify $\left(\frac{1+sin\frac{\pi}{8}+icos\frac{\pi}{8}}{1+sin\frac{\pi}{8}-icos\frac{\pi}{8}}\right)^8$	**7**
Q.5	**(a)**	Find the fourth root of -1	**3**
	(b)	Prove that $\sinh^{-1} x = log\{x + \sqrt{x^2+1}\}$.	**4**
	(c)	Prove that the nth root of unity are in geometric progression with the common ratio $\left(\cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}\right)$and show that the continued product of all n^{th} roots is $(-1)^{n+1}$.	**7**
		OR	
Q.5	**(a)**	Find the image of infinite strip $0 \le x \le 1$ under the transformation $w = iz + 1$. Sketch the region in the w-plane.	**3**
	(b)	Show that $u = x^2 - y^2 + x$ is harmonic, Find the corresponding analytic function $f(z) = u + iv$	**4**
	(c)	Using Cauchy's Integral formula, evaluate $\oint \frac{1}{z^2-7z+12}dz$, where C is the circle $\|z\| = 3.5$	**7**

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