GUJARAT TECHNOLOGICAL UNIVERSITY
BE – SEMESTER- VII EXAMINATION-SUMMER 2023
Subject Code: 3171611   **Date: 26/06/2023**
**Subject Name: Graph Theory and Combinatorics**
Time: 10:30 AM TO 01:00 PM   **Total Marks: 70**
Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

		MARKS

Q.1 **(a)** Define following terms: **03**
    i)   Connected Graph
    ii)   Closed walk
    iii)   Euler Trail

    (b) Give an example of a connected graph that has   **04**
    i)   Neither an Euler circuit nor a Hamilton Cycle.
    ii)   An Euler circuit but no Hamilton cycle.
    iii)   A Hamilton cycle but no Euler circuit.
    iv)   Both a Hamilton cycle and Euler circuit.

    (c) Define prefix code. Construct an optimal prefix code for the   **07**
symbols A, B, C, D, E, F, G, H, I, J that occur with respective
frequencies 78, 16, 30, 35, 125, 31, 20, 50, 80, 3.

Q.2 **(a)** Explain isomorphism of graphs. Determine whether or not the   **03**
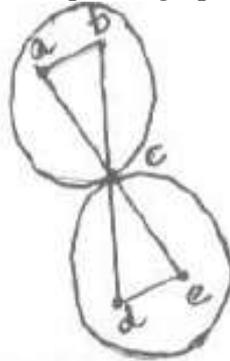following graphs are isomorphic.
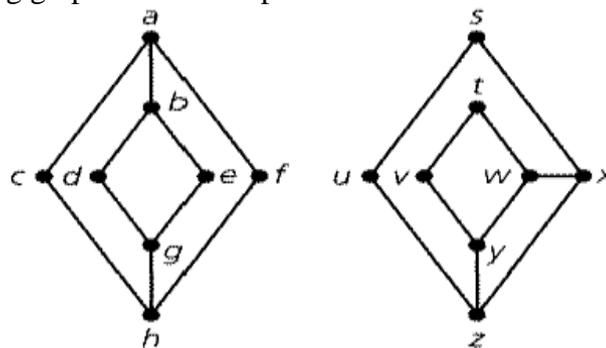
    (b) Describe The Seven Bridges of Konigsberg.   **04**
    (c) Define planar graph. If a connected planar graph G has n vertices,   **07**
e edges, r regions than prove that n – e + r = 2.
OR
    (c) Define chromatic number and chromatic polynomial. Determine   **07**
the chromatic polynomial for the graph shown in figure.

Q.3 **(a)** Find the dual for the given planar graph.   **03**

(b) Define a tree. Prove that tree with P vertices has P – 1 edges.   **04**
(c) State and prove Max – Flow and Min – cut theorem.   **07**

OR

Q.3 **(a)** Show that Peterson graph is a non planar graph.   **03**
(b) Define and give an example for each of the following: **04**
    i)   Complete Binary Tree
    ii)   Balanced Tree
(c) Find the maximum flow and the corresponding minimum cut for the transport network shown below: **07**
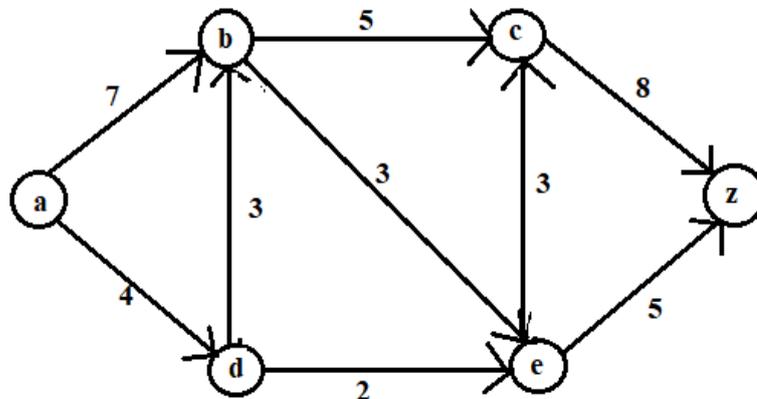
Q.4 **(a)** Determine the coefficient of a^5b^2 in expansion of $(2a - 3b)^7$.   **03**
(b) How many arrangements are there for all letters in the word SOCIOLOGICAL? In how many of these arrangements: **04**
    i)   A and G are adjacent
    ii)   All the vowels are adjacent.
(c) A student is to answer seven out of 10 questions on an examination. In how many ways can he make his selection if   **07**
    i)   There are no restrictions?
    ii)   He must answer the first two questions?
    iii)   He must answer at least four of the first six questions?

OR

Q.4 **(a)** Determine the coefficient of x^2yz^2 in expansion of $[(x/2) + y - 3z]^5$.   **03**
(b) How many derangements are there for 1, 2, 3, 4, 5 ?   **04**
(c) Determine the number of positive integers n, $1 \le n \le 2000$, those are   **07**
    i)   Not divisible by 2, 3 or 5
    ii)   Not divisible by 2, 3, 5 or 7

iii)   Not divisible by 2, 3 or 5 but are divisible by 7.

Q.5 **(a)** Find the exponential generating function for each of the following sequences: **03**
　　　i) $1, -1, 1, -1, 1, -1, \ldots\ldots$
　　　ii) $1, 2^2, 2^3, 2^4, \ldots\ldots$
　　　iii) $1, -a, a^2, -a^3, a^4, \ldots\ldots\ldots, a \in R$
(b) Find the coefficient of x^5 in $(1 - 2x)^{-7}$.   **04**
(c) Find generating function for the recurrence relation: **07**
$a_{n+2} - 3a_{n+1} + 2a_n = 0$, $n \geq 0$, with $a_0 = 1$, $a_1 = 6$.

OR

Q.5 **(a)** Solve the recurrence relation $a_n = 7a_{n-1}$, where $n \geq 1$ and $a_2 = 98$.   **03**
(b) Show that $(1 - 4x)^{-1/2}$ generates the sequence $\binom{2n}{n}$, $n \in N$.   **04**
(c) Define generating function. In how many ways can a police captain distribute 24 rifle shells to four police officers so that each officer gets at least three shells, but not more than eight?   **07**

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