Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–VI (NEW) EXAMINATION – SUMMER 2023

Subject Code:2160704 Date:10-07-2023

Subject Name:Theory of Computation

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** Define one-to-one function and also give its example. Decide and justify whether the function f : N → N defined by f (x) = min(x, 2) is one-to-one or not. **03**

(b) Let M_1 and M_2 be the FAs pictured in Fig. (i) and Fig. (ii) accepts the languages L_1 and L_2, respectively. **04**

Fig. (i)

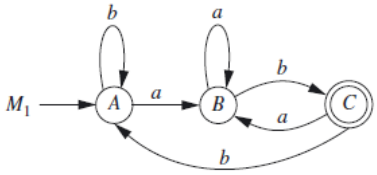
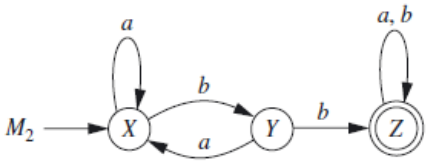


Fig. (ii)

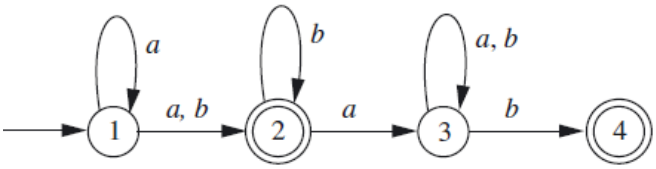


Draw FAs accepting the following languages:

(i) $L_1 \cap L_2$

(ii) L_1'

(c) Convert the following NFA into its equivalent DFA using the subset construction. **07**



Q.2 **(a)** Each case below gives a recursive definition of a subset L of $\{a, b\}^*$. Give a simple nonrecursive definition of L in each case. **03**

(i) a ∈ L; for any x ∈ L, xa and xb are in L.

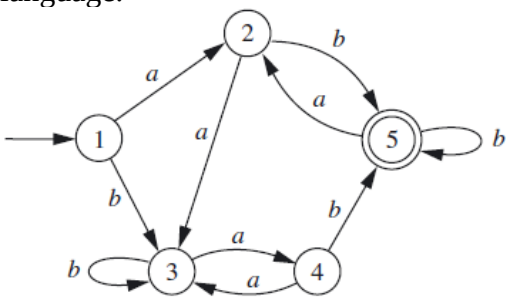
(ii) a ∈ L; for any x ∈ L, xb, ax, and bx are in L.

(b) Write CFG for generating each of the following languages: **04**

(i) a*b*

(ii) $\{a^i b^j c^k \mid j = i + k\}$

(c) Find a minimum-state FA for the following FA that recognizes the same language. **07**



OR

(c) Using the principle of mathematical induction, for all n ≥ 1, prove that, **07**

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots\ldots + n \times (n+1) = \frac{n\,(n+1)\,(n+2)}{3}$$

Q.3 **(a)** Check whether the following CFG is ambiguous or not with the help of word "abaa". **03**

S → a | Sa | bSS | SSb | SbS

(b) Using pumping lemma, prove that the language L = {$a^n b^n a\, b^{n+1}$: for all n = 1, 2, 3 …} is nonregular. **04**

(c) Define - Chomsky Normal Form. Also convert the following CFG into its equivalent CNF. **07**

S → XaX | bX | Y

X → XaX | XbX | ^

Y → ab

OR

Q.3 **(a)** Define - total language tree. Also find out the same for the following CFG: **03**

S → aa | bX | aXX

X → ab | b

(b) Define - pumping lemma for context-free languages. Also prove that the language L = {xx | x ∈ {a, b}*} is not CFL with the help of the same. **04**

(c) Define - Equivalence relation. Decide whether each of the following relations on the set {1, 2, 3, 4} is an equivalence relation or not. Justify your answer with proper reason. **07**

(i) R_1 = {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}

(ii) R_2 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}

Q.4 **(a)** Kill ^ - productions from the following CFG : **03**

S → AB | ^

A → aASb | a

B → bS

(b) Define - Pushdown Automaton and acceptance by a PDA **04**

(c) Convert the CFG with following productions into its equivalent PDA. Also trace out the same on input string abbaaa. **07**

S → a | aS | bSS | SSb | SbS

OR

Q.4 **(a)** Consider the CFG with productions: **03**

S → aA | bC | b

A → aS | bB

B → aC | bA | a

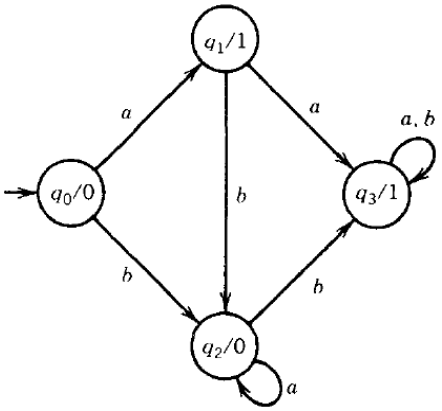
C → aB | bS

Does this CFG generate the language containing all the words with an even number of a's and an odd number of b's over Σ = {a, b}? Prove your answer.

(b) Explain Chomsky hierarchy in detail. **04**

(c) Design a PDA to accept strings with more a's than b's. Also trace out the same on input string abbabaa. **07**

Q.5 **(a)** Define - Moore machine. Convert the following Moore machine into its equivalent Mealy machine. **03**



(b) Define - Primitive Recursion. Also prove that the function, f : N → N defined by f (x, y) = x + y is primitive recursive. **04**

(c) Draw Turing machine for L = $\{a^n b^n c^n \mid n \geq 0\}$. Also trace out the same on input string abc. **07**

OR

Q.5 **(a)** Discuss - Context Sensitive Language with example. **03**

(b) Describe recursive languages and recursively enumerable languages. **04**

(c) Define Turing Machine. Also draw Turing machine that accepts the language of palindromes over {a, b}. **07**

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