Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– IV(NEW) EXAMINATION – SUMMER 2023

Subject Code:2141905 Date:07-07-2023

Subject Name:Complex Variables and Numerical Methods

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** **(i)** Find the real part of $(-1-i)^7+(-1+i)^7$ **[03]**

(ii) Is $Arg(z_1\ z_2)=Arg(z_1)+Arg(z_2)$? Justify your answer. **[04]**

(b) **(i)** Sketch the graph of the set $S=\{|\ z-1+2i\ |\le 2\}$ **[03]**

Does it define a domain?

(ii) If **[04]**

$$f(x,y)=\frac{x^2y}{x^4+y^2},\ if\ \ x^2+y^2\neq 0$$

$$=0,\ if\ \ (x,y)=(0,0)$$

Show that $\lim_{(x,y)\to\ (0,0)} f(x,y)$ does not exist.

Q.2 **(a)** **(i)** Check whether $f(z)=\sin z$ is analytic function or not. If so, find its derivative. **[03]**

(ii) Show that the real and imaginary parts of an analytic function are harmonic functions. **[04]**

(b) Evaluate $\int_C \mathrm{Re}(z^2)\,dz$ where c is a boundary of the square with vertices **[07]**

0, i, 1 + i, 1 in the clockwise direction.

OR

(b) Evaluate $\oint_C \frac{z-1}{(z+1)^2\ (z-2)}\,dz$ where c is the circle $|\ z-i\ |=2$ **[07]**

Q.3 **(a)** Discuss the convergence of $\sum_{n=0}^{\infty}\frac{(2n)!}{(n!)^2}(z-3i)^n$. **[07]**

Find also the radius of convergence.

(b) Define the Mobius transformation. Apply it to find the image of $|\ z\ |=1$ under **[07]**

$$w=\frac{i-z}{i+z}$$

OR

Q.3 **(a)** State Cauchy's residue theorem. Use it to evaluate $\oint_C \frac{z^2\sin z}{4z^2-1}dz$ where C is **[07]**

$|\ z\ |=2$

(b) Using Schwarz-Christoffel transformation, find the transformation which maps the angular region $0 \le \arg z \le \frac{\pi}{n}$ onto the half plane $v \ge 0$ and the points $z = 0$ and $z = 1$ onto $w = 0$ and $w = 1$ respectively in the w-plane. **[07]**

Q.4 **(a)** **(i)** Define forward, backward and central difference operators. **[03]**

(ii) Using Newton' forward interpolation formula, find y at x =1.5 from the data given below **[04]**

x	0	1	2	3	4
y	– 10	– 8	– 8	– 4	40

(b) Explain Gauss-Seidel method for solving a linear system of three equations in three unknowns x, y and z. **[07]**

Apply it to solve $2x + y + 54z = 110,\ 6x + 15y + 2z = 72,\quad 27x + 6y - z = 85$ correct to four decimal places.

OR

Q.4 **[07]**

(a) Evaluate $\int_{-2}^{6} \left(1 + x^2\right)^{\frac{3}{2}} dx$ by Gaussian quadrature formula for n = 3

(b) State Newton's divided difference formula. Apply it to find a polynomial of the degree three from the following data: **[07]**

x	1	2	7	8
y	1	5	5	4

Q.5 **(a)** **(i)** By power method, determine the largest eigen value of **[03]**

$$A = \begin{bmatrix} -1 & 1 & 4 \\ 10 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

(ii) Using secant method, find a real root of $x^3 - 2x - 5 = 0$ correct to three decimal places starting with $x_0 = 2$ and $x_1 = 3$ **[04]**

(b) Using Euler's formula, solve y for x = 0.1 from **[07]**

$$\frac{dy}{dx} = x + y + xy,\ \ y(0) = 1 \ and \ h = 0.02$$

OR

Q.5 **(a)** **(i)** Apply residue theorem to evaluate **[03]**

$$\int_0^{2\pi} \frac{4\, d\theta}{5 + 4\sin\theta}$$

(ii) Use bisection method to find first four iterations for **[04]**

$x^3 - 4x - 9 = 0$ taking $x_0 = 2.705$

(b) Using fourth order Runge-Kutta method, find y at x = 1 given that **[07]**

$$\frac{dy}{dx} = \frac{y - x}{y + x},\ \ y(0) = 1 \ and \ h = 0.5$$