Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II(NEW) EXAMINATION – SUMMER 2023

Subject Code:2110014 **Date:31-07-2023**

Subject Name:Calculus

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
1. **Make suitable assumptions wherever necessary.**
2. **Figures to the right indicate full marks.**
3. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **Objective Question (MCQ)** **Mark**

(a) **07**

1. The $\lim_{n\to\infty}\frac{2-3n^4}{n^4+1} =$ ______

(a) 1 (b) 2 (c) -3 (d) -1

2. The sum of the series $1+\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+ is.....$

(a) 3 (b) 5 (c) 11 (d) 9

3. If the equation of the curve contains only even powers of x and y then the curve is symmetric about…..

(a) X-axis (b) Y – axis (c) line Y=X (d) both X-axis and Y-axis

4. The $\lim_{x\to 1}(x-1)^{(x-1)} =$ ________

(a) 3 (b) 1 (c) 2 (d) 0

5. The value of $\int_1^\infty \frac{1}{x^2}dx$ is ……

(a) -2 (b) 5 (c) 1 (d) 3

6. When the area bounded by the curve y=f(x), the ordinates x=a, x=b and the x-axis is revolved about x-axis then the volume of the solid generated is given by …….

(a) $\pi\int_a^b ydx$ (b) $2\pi\int_a^b ydx$ (c) $\int_a^b y^2dx$ (d) $\pi\int_a^b y^2dx$

7. The value of $\lim_{(x,y)\to(5,1)}\frac{2xy}{x+y}$ is…….

(a) $\frac{5}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{4}{3}$

(b) **07**

1. If $f(x,y,z)=2x\sin(y+5z)\ then\ \frac{\partial f}{\partial z}\ is..$

(a) $10x\cos(y+5z)$ (b) $x\cos(y+5z)$

(c) $10\cos(y+5z)$ (d) $5x\cos(y+5z)$

2. If U is a homogeneous function of degree n in the variables x and y then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$ ______

(a) (n-1)u (b) (n+1) u (c) nu (d) (2n+1)u

3. $\int_2^4\int_1^2 6xy^2dydx=$ _____

(a) 35 (b) 84 (c) 20 (d) 45

4. Maclaurin's series of $e^{-x} is.....$

(a) $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$ (b) $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ (c) $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{2n!}$ (d) $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{(n+1)!}$

5. The degree of the homogeneous function

$f(x,y)=\frac{x+y}{\sqrt{x}+\sqrt{y}}\ is....$

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) 1 (d) $\frac{1}{2}$

6. What does the polar equation $r=a, a>0$ represent ?

(a) Line (b) circle (c) rectangle (d) Parabola

7. $\lim_{x\to 0}\left(\frac{1}{x^2}-\cos ec^2x\right)$ is of the form _________

(a) ∞^{∞} (b) $\frac{0}{0}$ (c) $\infty-\infty$ (d) 1^{∞}

Q.2 **(a)** Test the convergence of $\sum_{n=1}^{\infty} \tan^{-1} n - \tan^{-1}(n+1)$ **03**

(b) Find the values of a and b such that **04**

$$\lim_{x \to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$$

(c)

(1) If $u = \frac{x^2 + y^2}{\sqrt{x+y}}$ **then show that** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$ **03**

(2) If $u = \log(x^3 + y^3 - x^2 y - xy^2)$ **then show that** **04**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -3$$

Q.3 **(a)** Find the local extreme values of the function **03**

$$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$$

(b) $If\ \ y = f(x+ct) + g(x-ct)$ then prove that **04**

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

(c) **If** $u = f(x,y)\ \ where\ \ x = r\cos\theta, y = r\sin\theta$ **07**

then prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$

Q.4 **(a)** Evaluate $\iint_R (2x - y^2)dxdy$ where R is the triangular region R enclosed between the lines **03**

$$y = -x+1, y = x+1\ and\ \ y = 3.$$

(b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2 + y^2)dydx$ by changing to polar coordinates where a > 0. **04**

(c) Sketch the region and by changing the order of integration evaluate **07**

$$\int_0^1 \int_x^1 \sin y^2 dydx$$

Q.5 **(a)** Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2n-1}{n(n+1)(n+2)}$ **03**

(b) Test the convergence of **04**

(1) $\sum_{n=1}^{\infty} \frac{3n}{5n+1}$ (2) $\sum_{n=1}^{\infty} \left(\frac{1}{3^n} + \frac{1}{4^n}\right)$

	(c)	**Examine for which values of x the series** $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^{n+1}}{n+1}$	**07**
Q.6	**(a)**	If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$ then show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$	**03**
	(b)	Prove that $\int_{1}^{\infty}\frac{dx}{x^{p}}$ converges when $p \succ 1\ and\ diverges\ when\ p \le 1$	**04**
	(c)	**Expand** $\tan^{-1}(x+h)$ **in powers of h and hence find the value of** $\tan^{-1}(1.003)$	**07**
Q.7	**(a)**	Find the volume of solid of revolution obtained by rotating the area bounded below the lines $2x+3y=6$ in the first quadrant about the x-axis.	**03**
	(b)	Using slicing method find the volume of a solidball of radius a.	**04**
	(c)	Trace the curve $y^2(2a-x)=x^3$	**07**

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