Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–VII (NEW) EXAMINATION – SUMMER 2022

Subject Code:3171611 Date:08/06/2022

Subject Name:Graph Theory and Combinatorics

Time:02:30 PM TO 05:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 (a) (1) Let m be the number of odd degree vertices of a graph. Which of the following options are always true? **07**

(A) m is odd (B) m is a multiple of 4 (C) m is even (D) m is prime

(2) A simple graph with n vertices can have at most ---- edges.

(A) n (B) $\frac{n(n-1)}{2}$ (C) $\frac{n(n+1)}{2}$ (D) n^2

(3) Any two vertices of a tree are connected with --- path(s)

(A) exactly one (B) two different (C) no (D) many

(4) In a connected graph G a sub-graph H of G that includes all the vertices of G and is also a tree is known as ----

(A) surrounding tree (B) super tree (C) special tree (D) spanning tree

(5) Lily is trying to decide what to wear. She has shirts in the following colors: red, purple, and blue, and she has pants in the following colors: black and white. How many different outfits can Lily choose from (assuming she selects one shirt and one pair of pants)?

(A) 6 (B) 5 (C) 12 (D) 3

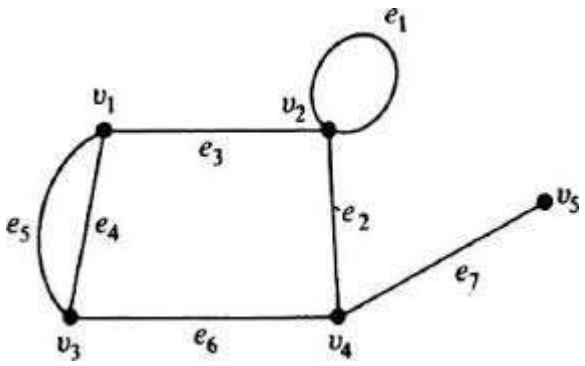
(6) In a survey of 1000 consumers it is found that 720 consumers likes product A and 450 liked product B. What is the least number that must have liked both the products?

(A) 70 (B) 170 (C) 270 (D) 1720

(7) The number of words, with or without meaning, that can be formed with the letters of the word ‘CHAIR’ are ---

(A) 20 (B) 40 (C) 120 (D) 5

(b)



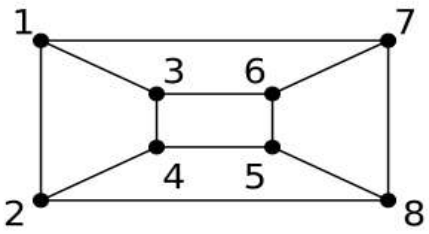
07

Answer the following questions for the given graph:

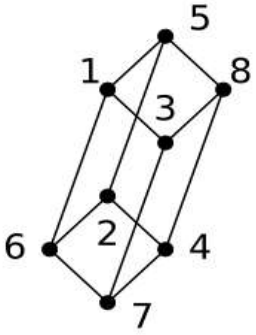
(1) How many edges are incident on vertex v_4?
(2) Name the parallel edges in the graph.
(3) Name the edge which is a self-loop.
(4) Name the pendant vertex.
(5) Verify the statement (in the graph given) " the sum of degrees of all vertices in a graph is twice the number of edges in the graph".

Q.2 **(a)** List five situations from everyday life in which graphs arise naturally and describe one of them. **07**

(b) (1) Check whether the below two graphs are isomorphic or not. **03**



(A)



(B)

(2) For the graph (A) above form two sets P and Q of vertices where one end of the edge is in the set P and other end of the edge is in the set Q. Write the name of such a graph. **04**

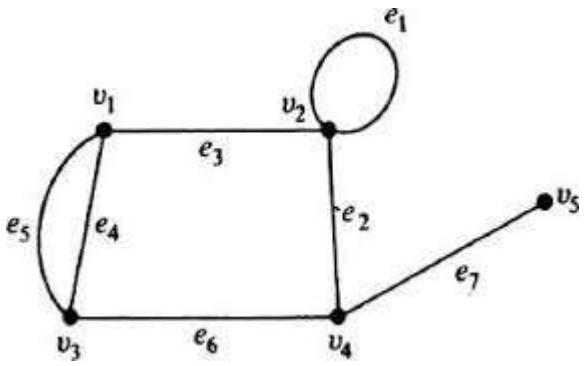
OR

(b) Prove the following:

(1) In any graph the sum of degrees of all vertices is twice the number of edges in the graph **03**

(2) In any graph, the number of vertices with odd degree is even. **04**

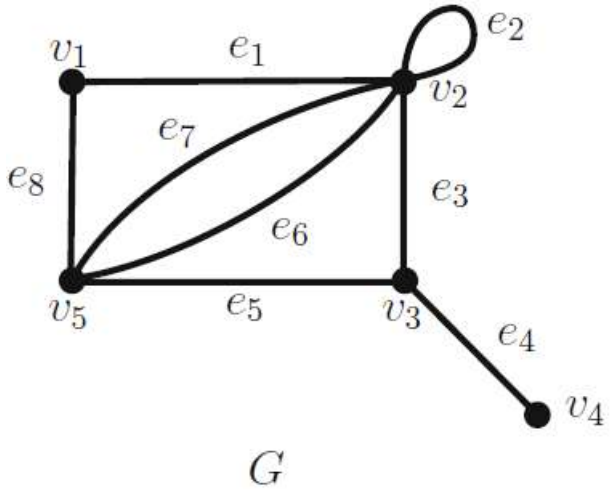
Q.3 **(a)** Obtain the incidence matrix and adjacency matrix of the following graph. **07**



(b)

(1) How many integers from 1 to 100 are multiples of 2 or 3? **03**

(2) For the following graph draw the graph **04**
(i) $G - \{v_2, v_5\}$ (ii) $G - \{e_2, e_4, e_7\}$



G

OR

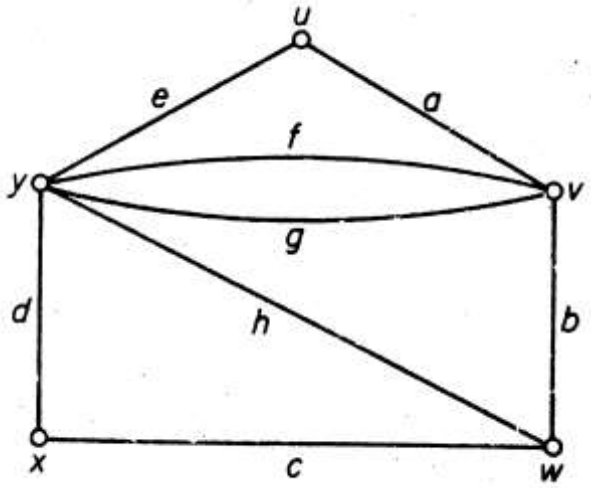
Q.3 (a)

- A complete bipartite graph is a simple bipartite graph with bipartition (X, Y) in which each vertex of X is joined to each vertex of Y; if $|X|$= m and $|Y|$ = n, such a graph is denoted by Km,n. **03**

(1) Draw the graph $K_{3,3}$.

- A *walk* in G is a finite non-null sequence W – $v_0e_1v_1e_2v_2 \ldots e_kv_k$ whose terms are alternately vertices and edges, such that, for $1 \leq i \leq k$, the ends of e_i are v_{i-1} and v_i.
- If the edges of a walk are distinct then it is called a *trail*.
- If the vertices of a walk are distinct then it is called a *path*.

(2) Obtain a walk, trail and a path of the graph given. **04**

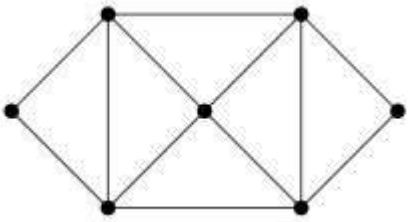


(b) (1) Draw a connected graph. Also draw a disconnected graph having three components. **04**

(2) In the graph of Que 3 (a) (2) obtain a closed walk and a cycle. **03**

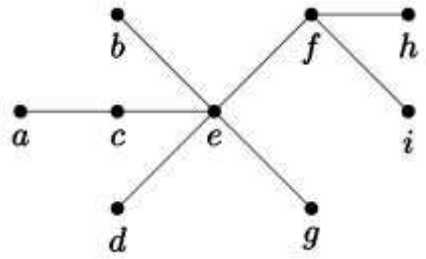
Q.4 (a) (1) Draw a tree having four vertices **01**

(2) Given a connected graph G, a *spanning tree* of G is a subgraph of G which is a tree and includes all the vertices of G. Draw a spanning tree for the following graph **03**



(3) Does the following graph a tree? **01**
G=(V,E) with V={a,b,c,d,e} and E={{a,b},{a,e},{b,c},{c,d},{d,e}.

(4) For the following tree if we designate vertex f as a root then mention the children of f. **02**



(b)

- The unique solution of the recurrence relation $a_{n+1} = da_n$ where $n \geq 0$, d is a constant and is given by $a_n = a_0 d^n$, $n \geq 0$.

(1) Find the unique solution of the recurrence relation $a_n = 7a_{n-1}$, where $n \geq 1$ and $a_2 = 98$. **03**

(2) A bank pays 6% annual interest on savings, compounding the interest monthly. If Vishal deposits ₹ 10,000 on the first day of May, how much this deposit be worth a year later? **04**

OR

Q.4 Prove the following:

1. If in a graph G there is one and only one path between every pair of vertices than graph G is a tree. **03**
2. A tree with n vertices has (n-1) edges. **04**
3. Any connected graph G with n vertices and (n-1) edges is a tree. **03**
4. A graph with n vertices, **(n-1)** edges and no circuit is a connected graph. **04**

Q.5 1. A college library has 40 textbooks on Sociology and 50 textbooks dealing with Anthropology. In how many ways a student of this college can select textbooks in order to learn more about one or the other of these two subjects? **14**

2. The drama club of Central University is holding tryouts for a spring play. With six men and eight women auditioning for the leading male and female roles. In how many ways director can cast his leading couple?

3. In how many ways the manufacture of license plates consisting of two letters followed by four digits (i) if no letter or digit repeated (ii) with repetition of letters and digits (iii) letters only vowel and digits even numbers only with repetition.

4. How many outcomes are possible when three dice are rolled, if no two of them may be the same?

5. Six people are to sit at a round table; how many seating arrangements are there?

6. How many ways are there to line up six people so that a particular pair of people is not adjacent?

7. How many permutations are there of the letters in Mississippi?

OR

Q.5 **(a)**

- The number of solutions to $\sum_{i=1}^{n} x_i = k$ is $\binom{k+n-1}{k}$, $x_i \geq 0$

1. Find the number of solutions to $x_1 + x_2 + x_3 + x_4 = 20$ with $x_1 \geq 0$, $x_2 \geq 1$, $x_3 \geq 2$ and $x_4 \geq -1$. **03**

- Second order linear homogeneous recurrence relation is $c_n a_n + c_{n-1} a_{n-1} + c_{n-2} a_{n-2} = 0, n \geq 0, c_n, c_{n-1}, c_{n-2}$ are real constants and $c_n \neq 0$. Solution to this is in the form $a_n = ck^n$ where $c, k \neq 0$ where k satisfies quadratic equation $c_n k^2 + c_{n-1} k + c_{n-2} = 0$. If two roots k_1, k_2 of equation are real and distinct then we take $a_n = A{k_1}^n + B{k_2}^n$ as general solution where A and B are arbitrary constants.

2. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ for $n \geq 2$ given that $a_0 = -1, a_1 = 8$. **04**

(b)

- Derangements: Nothing is in its right place. Number of derangements of n objects is $d_n = n!\left(1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots (-1)^n \frac{1}{n!}\right)$. For $n \geq 7$ a very good approximation of $\left(1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots (-1)^n \frac{1}{n!}\right) \approx \frac{1}{e} = 0.36786$

1. Obtain the derangements of numbers 1,2,3,…,10 **03**
2. Prove that the derangements of numbers 1, 2, 3, 4 are 9 and list them. **04**

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