Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2022

Subject Code:2141905 Date:23-06-2022

Subject Name:Complex Variables and Numerical Methods

Time:10:30 AM TO 01:30 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q-1	**(a)**	Solve the equation $z^2 - (5+i)z + 8 + i = 0$	**3**
	(b)	Show that $e^{\overline{z}}$ is nowhere analytic.	**4**
	(c)	Solve the following system of linear equations by Gauss-elimination method. $.x + y + z = 7, 3x + 3y + 4z = 24,\ \ 2x + y + 3z = 16$.	**7**
Q-2	**(a)**	Sketch $S = \{Z: -1 \le I_m(z) \le 2\}$. Is it connected?	**3**
	(b)	Show that $u = xsinx\ coshy - ycosx\ sinhy$ is harmonic.	**4**
	(c)	Find 5th roots of $-\sqrt{3} + i$.	**7**
		OR	
	(c)	Find an analytic function $f(z) = u + iv$, whose real part is $u = x^4 - 6x^2y^2 + y^4$. Also find the corresponding function v.	**7**
Q-3	**(a)**	Find the image of the infinite strip $0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$.	**3**
	(b)	Evaluate $\int_C (x^2 - iy^2)dz$ along the parabola $y = 2x^2$ from $(1,2)\ to\ (2,8)$.	**4**
	(c)	Evaluate $\oint_C \frac{e^z}{z(1-z)^3}$ dz, where C is (a) $\lvert z\rvert = \frac{1}{2}$ (b)) $\lvert z-1\rvert = \frac{1}{2}$	**7**
		OR	
Q-3	**(a)**	Show that $\frac{1+2z}{z^2+z^3} = \frac{1}{z^2} + \frac{1}{z} - 1 + z - z^2 + \cdots . in\ 0 < \lvert z\rvert < 1.$	**3**
	(b)	Find the bilinear transformation that maps respectively the points $i, 1, -i$ in z-plane onto the points $-i, 1, i$ in w-plane.	**4**
	(c)	Expand $f(z) = \frac{1}{(z+2)(z+4)}$ valid for region, $\lvert z\rvert < 2, 2 < \lvert z\rvert < 4\ and\ \lvert z\rvert > 4$.	**7**
Q-4	**(a)**	Given $\sin 45^\circ = 0.7071, \sin 50^\circ = 0.7660, \sin 55^\circ = 0.8192, \sin 60^\circ = 0.8660,$ find $\sin 52^\circ$, using Newton's forward interpolation formula.	**3**

(b) Using Newton-Raphson method, find the real root of $x^3 + x - 1 = 0$ correct upto 6 decimal places. **4**

(c) Solve the following equations by Gauss-Seidel method correct upto two decimal places. **7**

$20x + 2y + z = 30, x - 40y - 3z = -75, 2x - y + 10z = 30.$

OR

Q-4 **(a)** Evaluate $\int_0^1 \frac{dx}{1+x^2}$, using trapezoidal rule with $h = 0.2$. **3**

(b) Find the dominant eigen value of $A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$ by power method. **4**

(c) Using improved Euler's method, solve $y' = 1 - y$ with the initial condition $y(0) = 0$ and tabulate the solutions at $x = 0.1, 0.2$. Compare the answer with the exact solution. **7**

Q-5 **(a)** Write Gaussian one point, two point and three point formula **3**

(b) Solve $\frac{dy}{dx} = 3 + 2xy$ where $y(0) = 1\ for\ x = 0.1$ by Picard's method. **4**

(c) Use the Runge-Kutta method to solve $\frac{dy}{dx} = -xy^2\ for\ 0 \le x \le 1,$ subject to $y(0) = 2$. Use $h = 0.25$ and work for four decimal places. **7**

OR

Q-5 **(a)** Prove that :$\Delta = E - 1$, $\nabla = 1 - E^{-1}$. **3**

(b) Using Newton's divided difference interpolation formula compute $f(10.5)$ from the following data. **4**

x	10	11	13	17
f(x)	2.3026	2.3979	2.5649	2.8332

(c) The speed v of a car, after it starts is shown in the following table, v is in mt/sec and t is in seconds. **7**

t	0	12	24	36	48	60	72	84	96	108	120
v	**0**	**3.60**	**10.08**	**18.90**	**21.60**	**18.54**	**10.26**	**4.50**	**4.5**	**5.4**	**9.0**

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