Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2021

Subject Code:2141905 Date:03/09/2021

Subject Name:Complex Variables and Numerical Methods

Time:02:30 PM TO 05:30 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Separate real and imaginary parts of $\cosh z$	**03**
	(b)	Show that $f(z) = z^3$ is analytic everywhere.	**04**
	(c)	After applying partial pivoting to the following system of equations, use Gauss Jacobi method up to 3 iterations and find the approximate solution of the following system of equations. $x + 2y + 5z = 20$, $x + 4y + 2z = 15$, $5x + 2y + z = 12$	**07**
Q.2	**(a)**	Evaluate $\int_C \pi e^{\pi \bar{z}}\, dz$, where C is the line segment joining the points $1, 1+i$.	**03**
	(b)	Find the Bilinear Transformation which maps the points $z_1 = 1, z_2 = 0$ and $z_3 = -1$ of $z-$plane onto the points $w_1 = i, w_2 = \infty$ and $w_3 = 1$ of $w-$plane.	**04**
	(c)	Show that the transformation $w = iz + i$ maps the half plane $x > 0$ onto the half plane $v > 1$	**07**
		OR	
	(c)	Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$.	**07**
Q.3	**(a)**	Show that when $z \neq 0$, $\frac{e^z}{z^2} = \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + \frac{z^2}{4!} + \cdots$	**03**
	(b)	Find the center and radius of convergence of the power series $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2} z^n$	**04**
	(c)	Find the Laurent's series of the function $f(z) = \frac{1}{(z-2)(z-3)}$ in the regions (a) $2 < \lvert z \rvert < 3$ (b) $\lvert z \rvert > 3$	**07**
		OR	
Q.3	**(a)**	Find the $\int_C g(z)dz$ where, $C: \lvert z - i \rvert = 2$ and $g(z) = \frac{1}{(z-2i)}$.	**03**
	(b)	Find the $\int_C g(z)dz$, where $C: \lvert z \rvert = 2$ and $g(z) = \frac{1}{(z^2-5z+4)}$	**04**
	(c)	Using residue theorem, evaluate $\oint \frac{2z+6}{z^2+4}$ dz, where $C: \lvert z - i \rvert = 2$	**07**
Q.4	**(a)**	Find a root of the equation $x^3 - 4x - 9 = 0$ using the bisection method in 3 iterations.	**03**

	(b)	Expand $f(z) = \frac{1-\cos z}{z^2}$ in Laurent's series about $z = 0$ and identify the singularity.	**04**
	(c)	Use Newton's backward interpolation formula to find the polynomial which takes the following values: $y(0) = 1, \quad y(1) = 0, \ y(2) = 1, y(3) = 10$ And hence find $y(1.2)$	**07**

OR

Q.4	**(a)**	Find a root of $x^4 - x^3 + 10x + 7 = 0$ using Newton's Raphson method taking initial point -1.5.	**03**
	(b)	Solve the following system of equations by Gauss Elimination method. $x + y + z = 9$; $2x - 3y + 4z = 13$; $3x + 4y + 5z = 40$	**04**
	(c)	Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's $1/3^{\text{rd}}$ rule, taking $n = 6$. Hence, find $tan^{-1}6$.	**07**
Q.5	**(a)**	Use Euler's method solve for y at $x = 0.1$ from $\frac{dy}{dx} = x + y + xy, y(0) = 1$ in 3 steps	**03**
	(b)	Evaluate $\int_0^1 \frac{dt}{1+t}$ by Gaussian formula for n=3.	**04**
	(c)	Using the power method find the largest Eigen value for the matrix $A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$	**07**

OR

Q.5	**(a)**	Prove that $E = e^{hD}$	**03**
	(b)	Find a positive root of $xe^x - 2 = 0$ correct to two places of decimals by the method of False position, taking initial interval (0, 1).	**04**
	(c)	Apply Runge-Kutta fourth order method to find an approximate value of y when $x = 0.2$ in steps of 0.1 if $\frac{dy}{dx} = x + y^2$, given that $y = 1$ when $x = 0$.	**07**

****