Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– IV EXAMINATION – SUMMER 2020

Subject Code: 2141905 Date:28/10/2020

Subject Name: COMPLEX VARIABLES AND NUMERICAL METHODS

Time: 10:30 AM TO 01:30 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1	**(a)**	Verify Cauchy-Riemann equation for $f(z)=\cos x\cosh y - i\sin x\sinh y$.	**03**
	(b)	Find all cube roots of complex number $(-8i)$.	**04**
	(c)	Evaluate $\int_0^6 \frac{1}{1+x^2}dx$ with $h=1$ by (i) Trapezoidal Rule (ii) Simpson's $\frac{1}{3}$ Rule (iii) Simpson's $\frac{3}{8}$ Rule.	**07**
Q.2	**(a)**	Find the principal value of $(1-i)^{4i}$.	**03**
	(b)	Using Parametric representation of C, evaluate $\int_C \frac{z+2}{z}dz$; C is the circle $z=2e^{i\theta}\ (0\le\theta\le 2\pi)$	**04**
	(c)	Show that $u(x,y)=2x-x^3+3xy^2$ is harmonic function and find harmonic conjugate $v(x,y)$.	**07**
		OR	
	(c)	For $f(z)=\begin{cases}\frac{(\bar{z})^2}{z}; & z\neq 0\\ 0 \quad ; & z=0\end{cases}$, Show that C-R equation is satisfied at origin but $f'(0)$ does not exist.	**07**
Q.3	**(a)**	Derive the Taylor series representation $\frac{1}{1-z}=\sum_{n=0}^{\infty}\frac{(z-i)^n}{(1-i)^{n+1}};\quad \lvert z-i\rvert<\sqrt{2}$.	**03**
	(b)	State Cauchy Integral formula. Use it to evaluate $\int_C \frac{\cos z}{z(z^2+8)}dz$; C: $\lvert z\rvert=1$.	**04**
	(c)	Find Laurent series representation of $f(z)=\frac{1}{z(1+z^2)}$ for (i) $0<\lvert z\rvert<1$ (ii) $1<\lvert z\rvert<\infty$.	**07**
		OR	
Q.3	**(a)**	Determine residue of $f(z)=\frac{3z^3+2}{z^2+9}$ at $z=3i$.	**03**
	(b)	Find the fixed points of the transformation $w=\frac{z-1}{z+1}$.	**04**

(c) Evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ using residues. **07**

Q.4 **(a)** Show that $1+\Delta \equiv e^{hD}$ **03**

(b) Find $f(0.12)$ & $f(0.26)$ by appropriate interpolation formula from following table **04**

x	0.10	0.15	0.20	0.25	0.30
$f(x)$	0.1003	0.1511	0.2027	0.2553	0.3093

(c) Determine images of Vertical and Horizontal lines under the transformation $w = e^z$. **07**

OR

Q.4 **(a)** Using Lagrange's formula, express the function $\frac{3x^2+x+1}{(x-1)(x-2)(x-3)}$ as a sum of partial fractions. **03**

(b) Find interpolating polynomial using Newton's divided difference formula from following table **04**

x	0	1	4	5	7
$f(x)$	–6	–3	138	369	1611

(c) Show that a function $f(z) = u(x,y) + iv(x,y)$ is analytic in a domain D if and only if v is a harmonic conjugate of u. **07**

Q.5 **(a)** Use Newton-Raphson method to find positive root of $\sin x = 1 - x$ correct to three decimal places. **03**

(b) Use power method to find largest eigen value and corresponding eigen vector of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ correct to four decimal places. **04**

(c) Apply Runge-Kutta fourth-order method to find $y(0.2)$. Given that $dy/dx = y - x$ where $y(0) = 2$ and $h = 0.1$. **07**

OR

Q.5 **(a)** Use Secant method to find a positive root of the equation $x^3 + x - 1 = 0$ correct to three decimal places. **03**

(b) Given that $\frac{dy}{dx} = x^2 + y; \quad y(0) = 1$. Find $y(0.1)$ using Modified Euler's method with $h = 0.05$ correct to three decimal places. **04**

(c) Solve the following liner system **07**

$10x + 2y + z = 9$

$2x + 20y - 2z = -44$

$-2x + 3y + 10z = 22$

Correct to two decimal places by Gauss-Seidel method.

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