Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV(NEW) – EXAMINATION – SUMMER 2019

Subject Code:2141005 **Date:28/05/2019**

Subject Name: Signals and Systems

Time:02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Find whether given signal is periodic or not? If yes, give its fundamental period. **03**

(i) $x(t) = 3cos(10\pi t) + 5sin(6\pi t)$

(ii) $x[n] = e^{j10n}$

(b) Decompose following signals into their even and odd parts. **04**

(i) $x(t) = 3t^2 + 2t + 1$

(ii) $x[n] = \{1, \underset{\uparrow}{1}, 1\}$

(c) Explain following property for the system $y(t) = x(t) + 2$. **07**

(i) Linearity (ii) Time-invariance (iii) Causality (iv) Dynamicity (v) Stability.

Q.2 **(a)** Let $x[n]$ be a signal with $x[n] = 0$ for $n < -2$ and $n > 4$. For each of the following signal, determine the values of n for which it is guaranteed to be zero. **03**

(i) $x[n-3]$

(ii) $x[-n-2]$

(b) Prove associativity property of convolution sum. **04**

(c) For, **07**

$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$ and

$h[n] = 2\delta[n+1] + 2\delta[n-1]$,

Compute (i) $y_1[n] = x[n] * h[n]$ and (ii) $y_2[n] = x[n] * h[n+2]$.

OR

(c) Sketch each of the following signals for a signal shown in Figure 1. **07**

(i) $x(2-t)$

(ii) $x(2t+1)$

(iii)$[x(t) + x(-t)]u(t)$ [2+2+3 Marks]

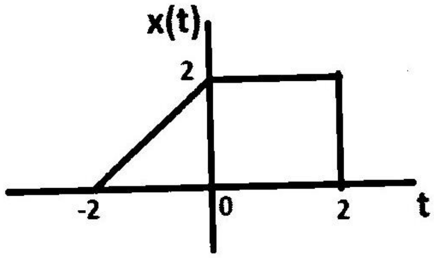

Figure 1

Q.3 **(a)** Find the convolution $x[n] * h[n]$, where $x[n] = \{1,2,3\}$ and $h[n] = \{1,\underset{\uparrow}{0},1\}$. **03**

(b) Given that $y[n] = x[n] * h[n]$, $x[n] = \{1,0,1\}$, $y[n] = 0 \; for \; n < -1$ and $y[n] = 2 \; for \; n = -1,0,3$. Find $h[n]$. Given $y[n]$ is of finite duration signal with length of 5. **04**

(c) Consider periodic signal $x(t)$ with fundamental frequency $\omega_0 = \pi$, determine its complex exponential Fourier series representation. Where, **07**

$$x(t) = \begin{cases} 1.5, & 0 \le t < 1 \\ -1.5, & 1 \le t < 2 \end{cases}$$

OR

Q.3 **(a)** Find the convolution $x(t) * h(t)$, where $x(t) = h(t) = e^{-at}u(t)$. **03**

(b) Given that $x[n]$ has Fourier transform $X(e^{j\omega})$, express the Fourier transform of the $w[n] = (n-1)^2 x[n]$ in terms of $X(e^{j\omega})$. [hint: Use Fourier transform property.] **04**

(c) Let $x(t)$ be a periodic signal with fundamental frequency ω_1 and Fourier coefficients a_k. Given that $y(t) = x(1-t) + x(t-1)$, how is the fundamental frequency ω_2 of $y(t)$ related to ω_1? Also, find a relationship between the Fourier series coefficients b_k of $y(t)$ and the coefficients a_k. **07**

Q.4 **(a)** State and prove Time scaling property of Fourier transform. **03**

(b) Given the relationships $y(t) = x(t) * h(t)$ and $g(t) = x(3t) * h(3t)$. Also given that $x(t)$ and $h(t)$ have Fourier transform $X(j\omega)$ and $H(j\omega)$ respectively. Using Fourier transform property show that $g(t) = Ay(Bt)$ and determine the values of A and B. **04**

(c) Find the response of an LTI system with impulse response $h(t) = e^{-at}u(t)$, $a > 0$ to the input signal $x(t) = e^{-bt}u(t)$, $b > 0, a \neq b$, using Fourier transform. **07**

OR

Q.4 **(a)** State and prove Duality property of Fourier transform. **03**

(b) A stable LTI system characterized by the differential equation **04**

$$\frac{dy(t)}{dt} + ay(t) = x(t), \qquad a > 0.$$

Find the impulse response of the system.

(c) Find the Fourier transform of $x(t) = e^{-|t|}$. Using property find Fourier transform of $\frac{2}{1+t^2}$. **07**

Q.5 **(a)** Find the inverse z-transform of $X(z) = z^{-2} + 1 + z^3, 0 < z < \infty$. **03**

(b) Discuss causality and stability of LTI system using z-transforms. **04**

(c) Using partial fraction expansion find the inverse z-transform of **07**

$$X(z) = \frac{1 - \frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})}, \qquad |z| > 2$$

OR

Q.5 **(a)** Find DTFT of $x[n] = \{1,0,4,2\}$. **03**

(b) State and prove differentiation property of z-transform. **04**

(c) Consider the following algebraic expression for the z-transform X(z) of signal $x[n]$: **07**

$$X(z) = \frac{1 + z^{-1}}{1 + \frac{1}{3}z^{-1}}$$

(i) Assuming the ROC to be $|z| > 1/3$, use long division to determine the values of $x[0]$, $x[1]$, and $x[2]$.

(ii) Assuming the ROC to be $|z| < 1/3$, use long division to determine the values of $x[0]$, $x[-1]$, and $x[-2]$.

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