GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER–IV (NEW) - EXAMINATION – SUMMER 2018

Subject Code:2141905   **Date:17/05/2018**
Subject Name:Complex Variables and Numerical Methods
Time:10:30 AM to 01:30 PM   **Total Marks: 70**
Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Separate real and imaginary parts of $f(z) = e^z$. Also prove that it is analytic everywhere.   **03**

(b) Write De Moivre's theorem. Find out cube root of i.   **04**

(c) Apply Gauss elimination method to solve system of equations: **07**
$6x + 2y + 8z = 26$
$3x + 5y + 2z = 8$
$8y + 2z = -7$

Q.2 (a) Evaluate $\oint_C \dfrac{e^{2z}}{(z - \ln 1.5)^3} dz$, where C is the square with vertices (1,0), (0,1), (0-1) and (-1,0)   **03**

(b) Verify whether $u(x, y) = x^2 - y^2$ is harmonic or not? Also find out its harmonic conjugate.   **04**

(c) State Cauchy – Riemann theorem. Write C-R equations in polar form and verify it for $f(z) = \dfrac{z}{\bar{z}}$ in polar form.   **07**

OR

(c) Evaluate $\int_C |z|^2 dz$ where C is the boundary of the square with vertices (0,0), (1,0), (1,1), (0,1).   **07**

Q.3 (a) Find the radius of convergence of the series $\displaystyle\sum_{2}^{\infty} \dfrac{z^n}{n(\log n)^3}$ **03**

(b) Evaluate $\oint_C \dfrac{z^2 + 1}{(z^2 - 1)} dz$, if C is the circle of unit radius with center (I) at $z = 1$ and (II) at $z = -1$.   **04**

(c) Evaluate following integrals using residue: **07**

(1) $\oint_C \dfrac{2z^2 + 3}{z(z+1)(z+2)} dz$, where $C: |z| = 1.6$

(2) $\displaystyle\int_{-\infty}^{\infty} \dfrac{3x + 2}{x(x-4)(x^2 + 9)} dx$

Q.3 **(a)** Find series expression for $f(z) = \tan^{-1} z$ at $z = 0$ **03**

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ in to the points $w = i, 0, 1$ **04**

(c) Find all Taylor and Laurent series of $f(z) = \dfrac{-2z+3}{z^2 - 3z + 2}$ with center 0. **07**

Q.4 **(a)** Evaluate $\int_3^8 x^2 \log x \, dx$ using Trapezoidal rule with step size $h = 1$ **03**

(b) Apply Lagrange's interpolation formula to evaluate $Y(3)$ using data given below: **04**

X	0	1	2	4
Y	-1	2	7	23

(c) Find the 3th root of the 119 correct up to five decimal places, using Newton Raphson method. **07**

OR

Q.4 **(a)** Evaluate $\int_0^1 \dfrac{dt}{1+t}$ using Two point Gaussian formula. **03**

(b) Use Bolzano method to find the positive root of $x = \cos x$ correct up to three decimal places. Use initial values 0.72 and 0.75 **04**

(c) Use appropriate Newton's formula to find the values of $Y(21)$ and $Y(28)$ from the data given below: **07**

X	20	23	26	29
Y	79.8	121.44	175.5	243.6

Q.5 **(a)** Apply Gauss Jacobi method to solve system of linear equation as under: **07**

$20x + 2y + z = 30$

$x - 40y - 3z = -75$

$2x - y + 10z = 30$

(b) Given that $\dfrac{dy}{dx} = 3x + y$ with $y(1) = 1.3$, consider step size $h = 0.1$. Find **07** $y(1.1)$ and $y(1.2)$ using Euler's method. Also find $y(1.3)$ using Runge- Kutta method of order 4.

OR

Q.5 **(a)** Determine the largest Eigen value and the corresponding eigen vector of the **07**

matrix $A = \begin{bmatrix} 4 & 4 & 2 \\ 4 & 4 & 1 \\ 2 & 1 & 8 \end{bmatrix}$

(b) Using Taylor's Series method solve $\dfrac{dy}{dx} = x^2 y - 1, \ y(0) = 1$. Also find **07** $y(0.03)$

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